

2. PROPERTIES OF STRUCTURE

The main structural properties are connected with *regularity* and *positions (symmetry)* of structure. Structural properties are read out from basic binary signs in the structure model **SM** [35].

2.1. Diversity of structural regularity

Propositions 2.1. On description the *regularities*:

P2.1.1. Graph, where the numbers of partial binary(+)signs $+d$ in all the rows i of **SM** are equal is (*degree*)-*regular*.

P2.1.2. Graph, where the partial signs $-d$ of all the binary(-)signs $-dnq$ in **SM** equal is d -*distance-regular*.

On example 2.1 showed Petersen graph with its pair(-)sign $-2.3.3$ is *2-distance-regular*.

P2.1.3. Graph, where the partial signs $+d$ of all the binary(+)signs $+dnq$ in **SM** are equal is $(d+1)$ -*girth-regular*.

For example, Petersen graph with its pair(+)sign $+4.10.15$ is *5-girth-regular* (example 2.1). In girth-regular graph belong all the n vertices the same number times to girth with length $n-a$.

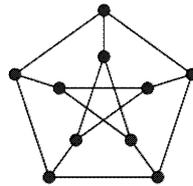
P2.1.4. Graph, where the numbers of clique signs $+(d=2).n.(q=n(n-1):2)$ in all the rows i of **SM** are equal is n -*clique-regular*.

For example, the complement of Petersen is *4-clique-regular* (example 2.1). If a *transitive* graph self not clique, then there can not exists a single clique, it can be only clique regular. In clique-regular graph belong all the n vertices the same number times to clique with power $n-b$.

P2.1.5. Graph said *strongly regular* with parameters (k,a,b) if it is a k -*degree-regular* incomplete connected graph such that any two adjacent vertices have exactly $a \geq 0$ common neighbors and any two non-adjacent vertices have $b \geq 1$ common neighbors.

For example, Petersen graph is *strongly regular*. Its strong regularity accrues from its bisymmetry (Prop. 2.11).

Example 2.1. Petersen graph *Pet*, the binary signs and structure model for Petersen graph and its complement *PetC* (the numbering starts here from the upper element and goes clockwise):



$A: -2.3.2; B: +4.10.15.$

$A: -2.6.12; B: +2.5.8.$

1 1 1 1 1 1 1 1 1 1		u_i
1 2 3 4 5 6 7 8 9 10	i	$AB \quad k$
0 B -A -A B B -A -A -A -A	1	63 1
0 B -A -A -A B -A -A -A	2	63 1
0 B -A -A -A B -A -A	3	63 1
0 B -A -A -A B -A	4	63 1
0 -A -A -A -A B	5	63 1
0 -A B B -A	6	63 1
0 -A B B	7	63 1
0 -A B	8	63 1
0 -A	9	63 1
0	10	63 1

1 1 1 1 1 1 1 1 1 1		u_i
1 2 3 4 5 6 7 8 9 10	AB	AB
0 -A B B -A -A B B B B	36	36
0 -A B B B -A B B B	36	36
0 -A B B B -A B B	36	36
0 -A B B B -A B	36	36
0 B B B B -A	36	36
0 B -A -A B	36	36
0 B -A -A	36	36
0 B -A	36	36
0 B	36	36
0	36	36

Explanations to show that it is possible to read out from the structure model:

- a) Petersen graph *Pet* is *3-degree-, 2-distance- and 5-girth-regular*.

- b) Binary sign **+4.10.15** means, that the element pair belongs to an assemblage of 5-girths, which consists of 10 elements and 15 connections (edges) – it is the *complete invariant* of Petersen graph, such sign do not have other structures.
- c) **Pet** is **5-girth-regular**, there exist twelve 5-girths, in present case: (1): 1-2-3-4-5-1, (2): 6-8-10-7-9-6, (3): 1-2-3-8-6-1, (4): 1-2-7-10-5-1, (5): 1-5-4-9-6-1, (6): 2-3-4-9-7-2, (7): 3-4-5-10-8-3, (8): 1-2-7-9-6-1, (9): 1-5-10-8-6-1, (10): 2-3-8-10-7-2, (11): 3-4-9-6-8-3, and (12): 4-5-10-7-9-4. Each element belong to six girths, each edge belongs to four girths.
- d) The **complement** of Petersen graph **PetC** is **4-clique-regular**. Explicit clique sign do not exist, but *binary graph* of binary sign **+2.5.8** contains the 4-clique. For example, the local structure model of binary graph with sign **+2.5.8** for elements 1 and 3 contains the signs of 4-clique, **+2.4.6**, that shows the existence of 4-clique 1,3,9,10:

$$-A: -2.4.5; \quad B: +2.3.3; \quad C: +2.4.6; \quad D: +2.5.8.$$

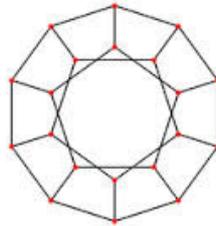
	1	3	9	10	7	<i>i</i>	ABCD	<i>k</i>	123
0	D	C	C	B		1	0121	1	121
0	C	C	B		3	0121	1	121	
0	C	-A			9	1030	2	210	
0	-A				10	1030	2	210	
0					7	2200	3	200	

- e) And so exists in the complement five intersected 4-cliques, in present case with elements: (1): 1,3,9,10; (2): 2,4,6,10; (3): 1,4,7,8; (4): 2,5,8,9; and (5): 3,5,6,7. Each element belongs to two cliques, each edge belongs to one clique.

Ideology of almost all the clique algorithms is oriented to recognition a single maximum clique. Such methods have been developed. Unfortunately, not have interest for the clique regularity.

Proposition 2.2. A transitive, i.e. vertex symmetric graph is *girth-* or/and *clique-regular*.

Example 2.2. Dodecahedron **Dod** and structure models dodecahedron and its complement **DodC**:



$$-A=-5.20.30; \quad -B=-4.8.9; \quad -C=-3.4.3; \quad -D=-2.3.2; \quad +E=+4.8.9.$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	<i>i</i>	ABCDE	<i>k</i>
0	E	-D	-C	-B	-C	-D	E	-D	-C	-B	-A	-B	-C	-D	-D	-C	-C	-D	E	1	13663	1
0	E	-D	-C	-C	-D	-D	-C	-B	-A	-B	-C	-D	-E	-D	-C	-B	-C	-D		2	13663	1
0	E	-D	-D	E	-D	-C	-C	-B	-C	-C	-D	-D	-C	-D	-C	-B	-A	-B	-C	3	13663	1
0	E	-D	-D	-C	-B	-C	-C	-D	-D	E	-D	-C	-C	-B	-A	-B	-A	-B		4	13663	1
0	E	-D	-C	-C	-D	-D	E	-D	-D	-C	-B	-C	-C	-B	-A	-B	-C	-B	-A	5	13663	1
0	E	-D	-D	E	-D	-D	-C	-C	-B	-A	-B	-C	-C	-B						6	13663	1
0	E	-D	-D	-C	-C	-B	-C	-C	-B	-A	-B	-C	-B	-A	-B	-C	-C	-C		7	13663	1
0	E	-D	-C	-B	-A	-B	-C	-C	-B	-C	-B	-C	-B	-C	-D	-D				8	13663	1
0	E	-D	-C	-B	-A	-B	-C	-C	-D	E	-D									9	13663	1
0	E	-D	-C	-B	-A	-B	-C	-D	-D	-C										10	13663	1
0	E	-D	-C	-B	-A	-B	-C	-D	E	-D	-C									11	13663	1
0	E	-D	-C	-C	-D	-D	-C	-B												12	13663	1
0	E	-D	-C	-C	-D	-D	-C	-B												13	13663	1
0	E	-D	-D	-C	-B	-C														14	13663	1
0	E	-D	-C	-C	-D															15	13663	1
0	E	-D	-D	E																16	13663	1
0	E	-D	-D																	17	13663	1
0	E	-D	-D																	18	13663	1
0	E	-D																		19	13663	1
0																				20	13663	1

Explanation: **Dod** is **3-degree-** and **5-girth regular**.

$$-A=-2.16.102; +B=+2.14.78; +C=+2.14.79; +D=+2.15.89.$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	<i>i</i>	<i>ABCCD</i>	<i>k</i>
0	-A	D	B	C1	B	D	-A	D	B	C1	C2	C1	B	D	D	B	B	D	-A	1	36316	1
	0	-A	D	B	B	D	D	B	C1	C2	C1	B	D	-A	D	B	C1	B	D	2	36316	1
		0	-A	D	D	-A	D	B	B	C1	B	B	D	D	B	C1	C2	C1	B	3	36316	1
			0	-A	D	D	B	C1	B	B	D	D	-A	D	B	B	C1	C2	C1	4	36316	1
				0	-A	D	B	B	D	D	-A	D	D	B	C1	B	B	C1	C2	5	36316	1
					0	-A	D	D	-A	D	D	B	B	C1	C2	C1	B	B	C1	6	36316	1
						0	-A	D	D	B	B	C1	B	B	C1	C2	C1	B	B	7	36316	1
							0	-A	D	B	C1	C2	C1	B	B	C1	B	D	D	8	36316	1
								0	-A	D	B	C1	C2	C1	B	B	D	-A	D	9	36316	1
									0	-A	D	B	C1	C2	C1	B	D	D	B	10	36316	1
										0	-A	D	B	C1	B	D	-A	D	B	11	36316	1
											0	-A	D	B	B	D	D	B	C1	12	36316	1
												0	-A	D	D	-A	D	B	B	13	36316	1
													0	-A	D	D	B	C1	B	14	36316	1
														0	-A	D	B	B	D	15	36316	1
															0	-A	D	D	-A	16	36316	1
																0	-A	D	D	17	36316	1
																	0	-A	D	18	36316	1
																		0	-A	19	36316	1
																			0	20	36316	1

Explanation: The complement *DodC* is **2-distance- and 16-degree regular**.

Is the complement *DodC* of 5-girth-regular dodecahedra *Dod* clique-regular?

Example 2.3. In complement *DodC* the explicit clique signs no exist, but in the processing the binary graphs g_{ij} , for example with signs $+B=+2.14.78$, obtained local sign matrices $S_{1,4}$, $S_{5,9}$, $S_{3,16}$, $S_{6,13}$ and $S_{5,8}$, contain **8-clique signs** $+2.8.28$. On the ground of such local sign matrices can be to recognize all the “hidden” **partial 8-cliques** of *DodC*:

<i>i=</i>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
I	•			•			•			•		•			•		•		•	
II		•			•		•		•		•			•			•			•
III	•		•			•			•			•		•		•		•		
IV		•		•		•		•			•		•			•			•	
V			•		•			•		•			•		•			•		•

Explanation: Thus, the complement *DodC* is **8-clique-regular**, where all five partial cliques are **intercrossed**, and where all the 10 intercrossing edges belong to binary orbit **C2**.

<i>i-j=</i>	1-12	2-11	3-18	4-19	5-20	6-16	7-17	8-13	9-14	10-15
Partial clique	I	II	III	I	II	III	I	IV	II	I
Partial clique	III	IV	V	IV	V	IV	II	V	III	V

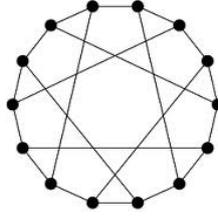
Form known graphs are *clique regular* also complements of Heawood’s, Coxeter’s, Folkman’s graphs. Their originals are bipartite and by all the nature laws represent the complements of such parts self-evidently cliques.

Proposition 2.3. Complement of a *m-partite* graph in case of equal *n* parts is ***n-clique regular***, with the number *m* of non-intercrossed ***n-cliques***.

Proposition 2.4. Partial cliques of a clique regular graph can be ***disconnected partial, mutually connected*** or ***intercrossed***.

Intercrossing can be exists on the aspect of vertices and edges. For example: cliques of *PetC* intercrossed by vertices, of *DodC* by edges.

Example 2.4. Heawood graph *Hea* (the numbering starts from the upper element and goes clockwise) the structure model of *Hea* and its complement *HeaC*:



$A: -3.8.9; B: -2.3.2; C: +5.14.21.$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	i	ABC	k	deg
0	+C	B	A	B	+C	B	A	B	A	B	A	B	+C	1	463	1	3
	0	+C	B	A	B	A	B	A	B	+C	B	A	B	2	463	1	3
		0	+C	B	A	B	+C	B	A	B	A	B	A	3	463	1	3
			0	+C	B	A	B	A	B	A	B	+C	B	4	463	1	3
				0	+C	B	A	B	+C	B	A	B	A	5	463	1	3
					0	+C	B	A	B	A	B	A	B	6	463	1	3
						0	+C	B	A	B	+C	B	A	7	463	1	3
							0	+C	B	A	B	A	B	8	463	1	3
								0	+C	B	A	B	+C	9	463	1	3
									0	+C	B	A	B	10	463	1	3
										0	+C	B	A	11	463	1	3
											0	+C	B	12	463	1	3
												0	+C	13	463	1	3
													0	14	463	1	3

$A: -2.10.36; B: +2.8.22; C: +2.9.30.$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	i	ABC	k	deg
0	A	+C	+B	+C	A	+C	+B	+C	+B	+C	+B	+C	A	1	346	1	10
	0	A	+C	+B	+C	+B	+C	+B	+C	A	+C	+B	+C	2	346	1	10
		0	A	+C	+B	+C	A	+C	+B	+C	+B	+C	+B	3	346	1	10
			0	A	+C	+B	+C	+B	+C	+B	+C	A	+C	4	346	1	10
				0	-A	+C	+B	+C	A	+C	+B	+C	+B	5	346	1	10
					0	A	+C	+B	+C	+B	+C	+B	+C	6	346	1	10
						0	A	+C	+B	+C	A	+C	+B	7	346	1	10
							0	A	+C	+B	+C	+B	+C	8	346	1	10
								0	-A	+C	+B	+C	A	9	346	1	10
									0	A	+C	+B	+C	10	346	1	10
										0	A	+C	+B	11	346	1	10
											0	A	+C	12	346	1	10
												0	A	13	346	1	10
													0	14	346	1	10

Explanations:

- Graph *Hea* is **3-degree-** and **6-girth-regular**. From 6-girth regularity concluded its **bipartite**, where its parts in present case divide to vertices with even numbers and vertices with odd numbers.
- Hea* is **structurally unique**, the pair sign **+5.14.21** signify that its 14 vertices form 21 adjacent pairs that belong to 6-girths and form a **complete invariant** of this graph.
- From bipartite *Hea* conclude that its complement *HeaC* consist of two **mutually connected 7-cliques**, it is **7-clique regular**, where the cliques correspond to the parts of *Hea*. *HeaC* is also **2-distance-** and **10-degree regular**.

2.2. Attributes of symmetry – positions

One of the key features of structure is **symmetry**. Symmetry is a structural characteristic that be expressed as a **recurrence** (in space or time) the similar parts (elements) of an object [20]. In this sense, represents symmetry an **equivalence class**, which consists of “similar” elements, and they form a **position** in the structure. Position characterized by **isomorphism of its accompanying graphs**.

However, it is a widespread understanding of *symmetry* characteristic, where the parts (elements) take similar then, if these are located from a central point, or an axis on the same distance [25]. Such widespread is in mathematics defined as: a) the shape feature “transform to itself” (e.g. isometric); b) feature of binary relation $xRy \leftrightarrow yRx$. Directed graphs called such a link (edge) as well as symmetrical.

Propositions 2.5. The relationships between *positions* and corresponding *subgraphs*.

P2.5.1. If vertices v_i, v_j, \dots have in graph G the same position ΩV_k then corresponding *subgraphs* $(G_i = G \setminus v_i) \cong (G_j = G \setminus v_j) \cong \dots$ are isomorphic.

P2.5.2. Edges $e_{ij}, e_{i^*j^*}, \dots$ have in graph G the same binary position ΩR_n then corresponding *subgraphs* $(G_{ij} = G \setminus e_{ij}) \cong (G_{i^*j^*} = G \setminus e_{i^*j^*}) \cong \dots$ are isomorphic .

General concept of symmetry is defined in mathematics by *automorphism* α , as a *substitution which retains the structure*. It is treated also as an *inner-* or *local isomorphism* (*isomorphism with itself*). Substitution has also been associated with renumbering of elements. In fact, none renumbering does not change the structure, it changes only the *removal, addition or relocation of a edge*. The automorphisms form the automorphism group of graph $AutG$ where its *transitivity domains* to *orbits* Ω called. An orbit is practically the same *equivalence class* what coincide whit previous *position class*. In case of $AutG$ be interested primarily on vertex orbits.

In the *frame of vertex positions* have also the *vertex pairs own positions* or *orbits*. Here is suitable issue from *binary positions* or *-orbits*. From now on we stay by term *position*.

Propositions 2.6. The relationships between *automorphisms, local isomorphisms, transitivity domains, binary positions* and *binary signs*:

P2.6.1. As an *automorphism* α or permutation that retain the structure be expressed in the form of a *local isomorphism* $G^{adj}_{ij} \cong G^{adj}_{i^*j^*}$ then constitute *transitivity domain of automorphisms* or *binary position* ΩR_n , i.e. an *isomorphism class of adjacent graphs* $\{G^{adj}_{ij1} \cong G^{adj}_{ij2} \cong \dots \cong G^{adj}_{ijq}\}_n \subseteq \Gamma_n^G$.

A binary position ΩR_n , as *isomorphism class* Γ_n can be interpretable also as an “isomorphism clique”, where all the element pairs are mutually isomorphic.

P2.6.2. Isomorphism class of adjacent graphs Γ_n^G is *replaceable* with corresponding *isomorphism class of binary graphs* $\{g_{ij1} \cong g_{ij2} \cong \dots \cong g_{ijq}\}_n \subseteq \Gamma_n^g$, that as well characterize the vertex pairs. Identification the elements of position ΩR_n take place by *binary signs* $\pm d.n.m.ij$ (or $\pm d.n.m.x.ij$) as the identifiers of isomorphism class Γ_n .

P2.6.3. In the model of structure SM is each vertex position ΩV_k related directly with binary positions ΩR_n of its incident edges.

Conclusions 2.1. Comparison the *group theoretic way of orbits recognition* and *semiotic modeling of positions*:

- 1) The *vertex-* ΩV_k and *binary positions* ΩR_n are recognized in structure model SM .
- 2) The orbits, recognized by group theoretic orbits, and positions, recognized by semiotic modelling, *coincide!*
- 3) Graphs with different structures can be have one and same group $AutG$, but have different semiotic models SM .
- 4) In case group theoretic treatment the number of permutations of completely symmetric graphs can be increase up to factorial. In case semiotic modelling of this does not happen.
- 5) In case group theoretic treatment the recognitions of vertex and edge orbits takes place separately and the “non-edge orbits” does not exist. In case semiotic modelling the recognitions of vertex-, pair(+)- and pair(-)orbits take place completely, where structure model SM express these in a complex.
- 6) Up to present considered, that orbit recognition belongs to periphery of graph theory. On the semiotic aspect it is a central problem.

Corollary 2.1. *A position (equivalence class) and orbit is the same.*

Symmetry properties desirable to be distinguish. Symmetry properties depend from the positions.

Definitions 2.1. The *symmetry kinds* of graph structure:

D2.1.1. Graph with only *one* vertex position ΩV_k we call *vertex symmetric* that also *transitive* called. For vertex symmetric or transitive graphs:

D2.1.2. Vertex symmetric graph with only *one* binary position ΩR_n^+ is *completely symmetric* or *complete graph*.

Empty graph with only one “non-edge” or pair position (orbit) is also *completely symmetric*.

D2.1.3. Vertex symmetric graph with only *one* edge position (i.e. binary(+)position) ΩR_n^+ and only *one* “non-edge” position (i.e. binary(-)position) ΩR_n^- we call *bisymmetric graph*.

For example, bisymmetric is Petersen graph (example 2.1).

D2.1.4. Vertex symmetric graph with *one* edge position (binary(+)position) ΩR_n^+ and *any* “non-edge” positions (binary(-)positions) ΩR_n^- we call *edge symmetric* or *(+)symmetric graph*.

For example, edge symmetric are here Dodecahedra (example 2.2) and Heawood’s (example 2.4) graphs. Complement of an *edge symmetric* graph is a “*non-edge*”- or *(-)symmetric graph*. Jointly we call these *mono symmetric graphs*.

D2.1.5. Vertex symmetric graph with *any* edge positions (binary(+)positions) ΩR_n^+ and *any* “non-edge” positions (pair(-)orbits) ΩR_n^- we call *poly-symmetric graph*.

For example, poly-symmetric are here the graphs on examples 3.2 and 3.8. Transitive graphs exist rarely. Among 156 of 6-elements structures exists such only 8.

For non vertex symmetric graphs:

D2.1.6. Graph with *more than one* vertex position ΩV_k , whereby at least to one ΩV_k belong at least two elements we call *partially symmetric graph*.

For example, partially symmetric graph showed on examples 1.2 – 1.5. Partial symmetry is a broad form of transition at symmetry to asymmetry. Among 156 of 6-elements structures are 140 partially symmetric.

D2.1.7. Graph where the number of vertices $|V|$ and vertex positions ΩV_k K is equal is a *0-symmetric* or *(completely) asymmetric graph*.

A 0-symmetric graph showed on example 1.8. In case of little graphs is also an exceptional phenomenon. For example, among 156 of 6-elements structures are 0-symmetric only 8.

For presentation the symmetry of structure is suitable to use corresponding signs.

Definitions 2.2. *Symmetry signs* of the structure:

D2.2.1 A vector with elements $|\Omega V|^m$, where $|\Omega V|$ is the power of a vertex position and m is the number of positions with such power, called *sign of vertex symmetry SVV*.

D2.2.2. A vector with elements $|\Omega R|^m$, where $|\Omega R|$ is the power of a pair position and m is the number of positions with such power, called *sign of pair symmetry SRV*.

Symmetry signs of the graphs on Examples 1.2 – 1.5 coincide, these are $SVV=1^1 2^1 3^1$, $SRV=1^1 2^1 3^2 6^1$. The *edge symmetry* is here different, on Example 1.2 it is $SEV=1^1 3^1 6^1$.

NB! Symmetry is *measurable*.

Propositions 2.7. *Measurement* of the symmetry. Symmetry signs give a good possibility to their *measuring*. To foundation of *symmetry size* is the classical Shannon’s formula of *information capacity*. Information capacity is practically a measure of *asymmetry* or *inner diversity*:

P2.7.1. *Vertex information capacity HV* depends from the number of vertices $|V|$ and the power of vertex positions $|\Omega V_k|$:

$$HV = -\sum_{k=1}^K PV_k \log PV_k$$

where $0 \leq PV_k = |\Omega V_k| : |V| \leq 1$.

There $\min HV = 0 \leq HV \leq \log |V| = \max HV$, where, if $K=1$, then $HV=0$ and if $K=|V|$, then $HV=\log |V|$.

P2.7.2. *Binary information capacity HR* depends from the number of vertex pairs $|R|$ and the power of binary positions $|\Omega R_n|$:

$$HR = -\sum_{n=1}^N PF_n \log PF_n$$

where $0 \leq PF_n = |\Omega R_n| : |R| \leq 1$ ja $|R| = [|V|(|V|-1)]:2$.

There $\min HR = 0 \leq HR \leq \log |R| = \max HR$, where, if $N=1$, then $HR=0$ and if $N=|R|$, then $HR=\log |R|$. Edge info capacity HR^+ calculates by the number of edges $|E|$ and the power of edge positions $|\Omega R_n^+|$. “Non-edge”

info capacity HR^- calculate by the number of “non-edges” $|R^-|$ and the power of corresponding pair positions $|\Omega R_n^-|$.

Information comes into being on the ground of certain *diversity*, *i.e. inner distinctness*. Information capacity depends from *quantity of variances*. There where variances no exist, arises a certain “domain of equability”, what on the structural aspect a *symmetry class* or *position* mean. Then more exist “domains of equability” or positions, then larger is information capacity HR and then smaller is the *symmetry size* (*value*).

Proposition 2.8. On the ground of information capacities HV and HR can be recognize the *symmetry values* SV and SR correspondingly:

$$SR = 1 - (HR: \log|R|), \text{ where } 0 \leq SR \leq 1.$$

The symmetry *value is 1*, if there exist *only one position*; the *value is 0*, if the *number of positions equal to the number of elements*. This give rise to *compare, order and grouping* the graphs with different size by symmetry values By analogy with the value of *banary symmetry* SR can be express the *binary(+)*symmetry (“*edge-symmetry*”) SE . Symmetry value SR is officially called as *regularity*.

Example 2.5. Symmetry-vectors and the symmetry-values of the graphs showed on various examples. Ordered by lessen of the edge symmetry SE :

Exm	Sym	K	N	SVV	SV	SRV	HR	SR	SEV	SE	aut	3003PS
2.7.	Bis	1	2	6 ¹	1.000	3 ¹ 12 ¹	0.722	0.815	3 ¹	1.000	48	99
2.8.	Bis	1	2	6 ¹	1.000	6 ¹ 9 ¹	0.971	0.751	6 ¹	1.000	72	6
1.5.	Prt	3	5	1 ¹ 2 ¹ 3 ¹	0.478	1 ¹ 2 ¹ 3 ² 6 ¹	2.106	0.461	3 ¹	1.000	12	396
1.4.	Prt	3	5	1 ¹ 2 ¹ 3 ¹	0.478	1 ¹ 2 ¹ 3 ² 6 ¹	2.106	0.461	1 ¹ 6 ¹	0.789	12	28
1.2.	Prt	3	5	1 ¹ 2 ¹ 3 ¹	0.478	1 ¹ 2 ¹ 3 ² 6 ¹	2.106	0.461	1 ¹ 3 ¹ 6 ¹	0.610	12	60
1.3.	Prt	3	5	1 ¹ 2 ¹ 3 ¹	0.478	1 ¹ 2 ¹ 3 ² 6 ¹	2.106	0.461	2 ¹ 3 ¹	0.582	12	60
4.2.	Prt	4	9	1 ² 2 ²	0.266	1 ³ 2 ⁶	3.107	0.205	1 ² 2 ⁴	0.241	2	360
GS76	0-sy	6	15	1 ⁶	0	1 ¹⁵	3.907	0	1 ⁸	0	1	336

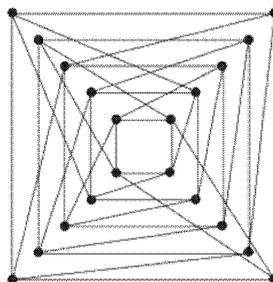
*

Each position is “naturalizable” in the form of a *position structure* [31, 35, 36].

Definition 2.3. *Position structure* GS_n is a structure that consists of element pairs, which belong to a certain *binary position* ΩR_n . The number of position structure equal to the number of binary positions.

The position structures opens some various “hidden sides” of the structure, that sometimes also “mystical” seems. In principle, the position structures are inevitable, so as the covering, cliques and others structural attributes, where their identification to a very practical and necessary deemed.

Example 2.6. *Bipartite and semi-symmetric* Folkman’s graph Fol , its binary signs, structure model and list of its position structures GS_n :



A: -4.14.21; B: -3.8.10; C: -2.6.8; D: -2.4.4; E: -2.3.2; F: +3.6.8.

																				u_i	k	s_i											
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1	2	3	4	5	6	7	8	9	10	i	ABCDEF		12
0	-E	-C	F	-B	-B	F	-B	F	-B	-B	-B	-F	11	061084	1	04																	
	0	-E	-E	-E	-E	-E	-E	-C	-E	-B	F	-B	-B	F	-B	-B	-B	-B	F	F	12	061084	1	04									
		0	-E	-E	-E	-E	-C	-E	-E	F	-B	F	-B	-B	-B	-B	-B	-B	F	F	B	13	061084	1	04								
			0	-E	-E	-C	-E	-E	-E	-B	F	-B	F	-B	-B	F	F	-B	-B	14	061084	1	04										
				0	-C	-E	-E	-E	-E	-B	-B	F	-B	F	F	F	-B	-B	-B	15	061084	1	04										
					0	-E	-E	-E	-E	-B	-B	F	-B	F	F	F	-B	-B	-B	16	061084	1	04										
						0	-E	-E	-E	-B	F	-B	F	-B	-B	F	F	-B	-B	17	061084	1	04										
							0	-E	-E	F	-B	F	-B	-B	-B	-B	F	F	-B	18	061084	1	04										
								0	-E	-B	F	-B	-B	F	-B	-B	-B	F	F	19	061084	1	04										
									0	F	-B	-B	F	-B	F	-B	-B	-B	F	20	061084	1	04										
										0	-A	-D	-D	-A	-D	-A	-D	-D	-D	1	360604	2	40										
											0	-A	-D	-D	-A	-D	-D	-D	-D	2	360604	2	40										
												0	-A	-D	-D	-D	-D	-A	-D	3	360604	2	40										
													0	-A	-D	-D	-D	-A	-D	4	360604	2	40										
														0	-D	-D	-A	-D	-D	5	360604	2	40										
															0	-D	-A	-A	-D	6	360604	2	40										
																0	-D	-A	-A	7	360604	2	40										
																	0	-D	-A	8	360604	2	40										
																		0	-D	9	360604	2	40										
																			0	10	360604	2	40										

Explanations:

- Graph Fol decompose correspondingly it's binary positions $-A$, $-B$, $-C$, $-D$, $-E$ and F to six position-structures:
- To binary position $-A$ corresponds position structure $Fol_{n=-A}$ is *Petersen's graph* (!). This fact is showed also in partial model $SM_{2,2}$, if there the sign $-A$ replaced with Petersen sign $+4.10.15$ and $-D$ replaced with sign $-2.3.2$ then it is equivalent with structure model of Petersen graph (see example 1.2).
- To binary position $-B$ corresponds position structure $Fol_{n=-B}$ turns out to *another semi-symmetric graph*, designed by V. Titov [41] that has also a position structure in the form of *Petersen graph*.
- To binary position $-C$ corresponds position structure $Fol_{n=-C}$ is a graph with ten components of 2-cliques.
- To binary position $-D$ corresponds position structure $Fol_{n=-D}$ is the *complement of Petersen graph* (!).
- To binary position $-E$ corresponds position structure $Fol_{n=-E}$ is the *complement of position structure $Fol_{n=-C}$* , i.e. *2-quinta clique*.
- To binary position $+F$ corresponds position structure $Fol_{n=+F}$ is naturally *Folkman graph* self.

The importance of position structures lies in the explaining structural properties, where these also recognize the identical particles of various structures. For example, could be argued that the semi-symmetrical graphs with 20 elements represent a kind of "genetic group" that contains position structures in the form of Petersen graphs. Also, all the graphs of n -polygons are proven to be the widespread position structures. Such relationships between the position structures appear in various ways. If the structure is divided to certain parts, or contain components, cliques, girths, etc., then appear the corresponding attributes in position structures in another forms.

Position structures GS_n opens the different "hidden" sides and particles of its initial structure GS . If the basic structure is partite or contains other components, as cliques, girths etc, then emerge the corresponding vertex complexes in the position structures in another form.

Propositions 2.9. Properties of position structures:

P2.9.1. Position structure is *element symmetric*, i.e. its elements belong to the same position $\Omega V_{k=1}$.

P2.9.2. To the binary(+)position ΩR_n^+ corresponds a position(+)structure GS_n^+ is a *partial structure* of GS ; to the binary(-)position ΩR_n^- corresponds a position(-)structure GS_n^- is a *partial structure* of complement $\lceil GS$.

P2.9.3. To each binary(+)position ΩR_n^+ of structure GS corresponds the binary(-)position of complement $\lceil GS$ where their *position structures coincides*, $GS_n^+ \equiv \lceil GS_n^-$.

- P2.9.4.** Some position structure GS_n can be appear isomorphic with initial structure, GS , $GS_n \cong GS$ (for example, a position structure of the cube is also cube).
- P2.9.5.** Different position structures GS_n of initial structure GS or position structures of different structures can be isomorphic or coincides.

By help position structures can be find the same attributes of various structures. For example, Hypercube and Möbius-Kantor graph have some common position structures etc. Under the looking are also the position structures of position structures, i.e. second and high degree position structures.

Propositions 2.10. Properties of high degree position structures:

- P2.10.1.** A second or high degree position structure can be isomorphic or coincides with a lower degree position structure or initial structure. Coincidence of a position structure and initial structure constitutes a reconstruction of initial structure.
- P2.10.2.** High degree position structures no open more complementary “hidden sides”, these begin to repeat.
- P2.10.3.** Formation of high degree position structure is a convergent process, it finished with a crop up or reconstruction a low degree or initial structure.

2.3. Relationships between regularity and symmetry properties

Interest for the relationship of regularity and symmetry is not seen, as in case of practical tasks these properties are usually not visible. However, it is with real legitimacy. An interesting relationship is between the bisymmetry and strong regularity, which seems to be “hidden” behind.

A graph said **strongly regular** with parameters (k, a, b) if it is a k -regular incomplete and connected graph such that any two adjacent vertices have exactly $a \geq 0$ common neighbors and any two non-adjacent vertices have $b \geq 1$ common neighbors.

Existence in connected bisymmetric structure exactly two different binary signs, $-d.n_1.q$ and $+d.n_2.q$, mean that by $\pm d=2$ has each nonadjacent vertex pair exactly n_1-2 common neighbors and each adjacent vertex pair n_2-2 common neighbors. In case $+d>2$ no exist common neighbors. The numbers $n-2$ of common neighbors can be stay constant also by existence more that two binary signs, i.e. by mono-, poly- and partial symmetries. Consequently, strongly regular graphs can be also mono-, poly- and partial symmetric (example 3.1).

Proposition 2.11. All the connected bisymmetric structures are **strongly regular** as well **girth-** or **clique regular**.

It has often unnoticed, the following fact.

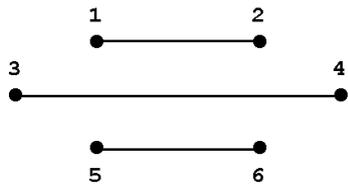
Proposition 2.12. The **connected complement** of a strongly regular structure is also **strongly regular**.

Proposition 2.13. The **complement** of a graph with m equal disconnected partial cliques is a **bisymmetric m -partite complete graph**, i.e. it is a **$n-m$ -clique** – and contrariwise.

Size	Graph	Its complement
m	Number of disconnected partial cliques	Number of parts
n	Power of disconnected partial cliques	Power of parts

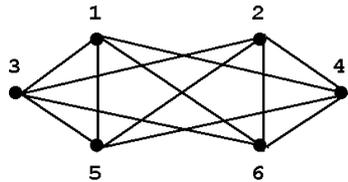
This mean that the complement of the structure with two disconnected partial cliques is a **bi-clique**, with three disconnected partial cliques is a **tri-clique**, etc. Not forget that the **complement** of bisymmetric graph is also bisymmetric.

Example 2.7. Graph **B6-3**, its complement **B6-12**, their binary signs, structure models and measures:



$$A: -0.2.0; B: +1.2.1.$$

	1	2	3	4	5	6	i	AB	deg
	0	B	-A	-A	-A	-A	1	41	1
		0	B	-A	-A	-A	2	41	1
			0	B	-A	-A	3	41	1
				0	B	-A	4	41	1
					0	B	5	41	1
						0	6	41	1



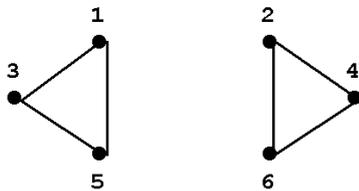
$$A: -2.6.12; B: +2.4.5.$$

	1	2	3	4	5	6	i	AB	deg
	0	-A	B	B	B	B	1	14	4
		0	-A	B	B	B	2	14	4
			0	-A	B	B	3	14	4
				0	-A	B	4	14	4
					0	-A	5	14	4
						0	6	14	4

SRV	HR	SR	aut
3 ¹ 12 ¹	0.2173	0.8152	48

Explanations: a) Graph **B6-3** and its complement **B6-12** are *bisymmetric*. b) Graph **B6-3** consist of *three disconnected partial 2-cliques*, it is *2-clique regular*. c) Complement **B6-12** is *three partite*, where its parts correspond to 2-cliques of **B6-3**. It is a so called *partite clique*, exactly with a *2-tri-clique*, generally called *n-m-clique*. It is simply sight that all the vertices belong to *triangles*.

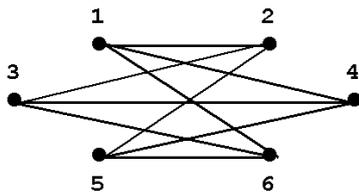
Example 2.8. Graph **B6-6**, its complement **B6-9**, their binary signs, structure models and measures:



$$A: -0.2.0; B: +2.3.3.$$

	1	2	3	4	5	6	i	ABC	deg
	0	-A	B	-A	B	-A	1	32	2
		0	-A	B	-A	B	2	32	2
			0	-A	B	-A	3	32	2
				0	-A	B	4	32	2
					0	-A	5	32	2
						0	6	32	2

$$A: -2.5.6; B: +3.6.9.$$



	1	2	3	4	5	6	i	AB	deg
	0	B	-A	B	-A	B	1	23	3
		0	B	-A	B	-A	2	23	3
			0	B	-A	B	3	23	3
				0	B	-A	4	23	3
					0	B	5	23	3
						0	6	23	3

SRV	HR	SR	aut
6 ¹ 9 ¹	0.2923	0.7515	72

Explanations: a) Graph **B6-6** and its complement **B6-9** are *bisymmetric*. b) Graph **B6-6** consist of *two disconnected partial 3-cliques*, it is *3-clique regular*. c) Complement **B6-9** is *bipartite*, where its parts correspond to 3-cliques of **B6-6**. d) **B6-9** is a *3-bi-clique* that is *4-girth regular*. e) Binary sign $+3.6.9$ cover all the $n=6$ vertices and all the $q=9$ edges, it is the *complete invariant* of **B6-9**.

Propositions 2.14. On *n-m-cliques* and *bisymmetry*.

P2.14.1. All the *n-m-cliques* with equal power n and their complements are *bisymmetric*.

Comment: Bisymmetric can be connected or disconnected.

P2.14.2. A *n-m-clique* contain an (usual)clique with the power m , it is *m-clique regular*.

Comment: For example, bi-clique is *2-clique regular*, tri-clique is *3-clique regular*, etc.

P2.14.3. A bisymmetric *n-m-clique* contain $s=n^m$ usual cliques with power *m*.

P2.14.4. The number of edges $|E|$ of bisymmetric *n-m-clique* equal to the product of quadrate of power n^2 of parts and the number *m* of edges in the usual clique:

<i>n-m-clique</i>			
Symmetry	Power of cliques	Number of cliques	Number of edges
<i>Bisymmetry</i>	<i>m</i>	n^m	$E=n^2m(m-1):2$

Proposition 2.15. Correspondingly to the number of parts we call a *n-m-clique* to *bi-, tri-, quadro-, Squinta-, sexta-, septa-, octa-, nona-, deca-, or undeca-* etc *-clique*.

Comments: Among the graphs with 1 to 20 vertices there exist exactly following *n-m-cliques*:

- One *n-m-clique* with 4-vertces – **2-biclique** as a complement of disconnected partial 2-cliques;
- Two *n-m-cliques* with 6 vertices – **2-tri-clique** and **3-bi-clique** as complements of disconnected partial 2- and 3-cliques correspondingly;
- Two *n-m-cliques* with 8 vertices – **2-quadro-clique** and **4-bi-clique** as complements of disconnected partial 2- and 4-cliques correspondingly;
- One *n-m-clique* with 9 vertices – **3-tri-clique** as a complement of disconnected partial 3-cliques correspondingly;
- Two *n-m-cliques* with 10 vertices – **2-quinta-** and **5-bi-clique** as complements of disconnected partial 2- and 5-cliques correspondingly;
- Four *n-m-cliques* with 12 vertices – **2-sexta-, 3-quadro-, 4-tri-** and **6-bi-clique** as complements of disconnected partial 2-, 3-, 4- and 6-cliques correspondingly;
- Two *n-m-cliques* with 14 vertices – **2-septa-** and **7-bi-clique** as complements of disconnected partial 2- and 7-cliques correspondingly;
- Two *n-m-cliques* with 15 vertices – **3-quinta-** and **5-tri-clique** as complements of disconnected partial 3- and 5-cliques correspondingly;
- Three *n-m-cliques* with 16 vertices – **2-octa-, 4-quadro-** and **8-bi-clique** as complements of disconnected partial 2-, 4- and 8-cliques correspondingly;
- Four *n-m-cliques* with 18 vertices – **2-nona-, 3-sexta-, 6-tri-** and **9-bi-clique** as complements of disconnected partial 2-, 3-, 6- and 9-cliques correspondingly;
- Four *n-m-cliques* with 20 vertices – **2-deca-, 4-quinta-, 5-quadro-** and **10-bi-clique** as complements of disconnected partial 2-, 4-, 5- and 10-cliques correspondingly;

In all **27 n-m-cliques**.

Conclusion 2.2. All the *n-m-cliques* are *strongly regular*, but no on the contrary.

In addition of simply constructable *n-m-cliques* are recognized following bisymmetric-strongly regular structures: **1)** self-complemented 5-girth; **2)** self-complemented B9-18; **3)** Petersen graph *PET* (B10-15); **4)** and its complement *PETC* (B10-30); **5)** self-complemented B13-39; **6)** Weisfeiler’s B15-45; **7)** and its complement B15-60; **8)** Greenwood’s (Clebish’s) B16-40; **9)** and its complement B16-80; **10)** Shrikhande B16-48; **11)** and its complement B16-72; **12)** self-complemented B17-68.

Consequently, among the structures with 1 to 20 vertices there exists $27+12=39$ bisymmetric + strongly regular + clique- or girth regular graphs

Example 2.9. All the *bisymmetric + strongly regular + clique- or girth regular* graphs with to 20 vertices:

Nr	Notation	deg	SRV	SR	Cmp/prt		Regularity	Number s	Commentary	Pair signs	
					m	n				Binary(-)sign	Binary(+sign)
1	B4-4	2	$2^1 4^1$	<i>0.6448</i>	2p	2	4-girth	-	2-bi-clique	<i>-2.4.4</i>	<i>+3.4.4</i>
2	B5-5	2	5^2	<i>0.6990</i>	1c	5	5-girth	-	Selfcomplem.	<i>-2.3.2</i>	<i>+4.5.5</i>
3	B6-12	4	$3^1 12^1$	<i>0.8152</i>	3p	2	3-clique	8	2-tri-clique	<i>-2.6.12</i>	<i>+2.4.5</i>
4	B6-9	3	$6^1 9^1$	<i>0.7515</i>	2p	3	4-girth	-	3-bi-clique	<i>-2.5.6</i>	<i>+3.6.9</i>
5	B8-24	6	$4^1 24^1$	<i>0.8769</i>	4p	2	4-clique	16	2-quadro-clique	<i>-2.8.24</i>	<i>+2.6.13</i>

6	B8-16	4	$12^1 16^1$	<i>0.7906</i>	2p	4	4-girth	-	<i>4-bi-clique</i>	-2.6.8	<u>+3.8.16</u>
7	B9-27	6	$9^1 27^1$	<i>0.8431</i>	3p	3	3-clique	27	<i>3-tri-clique</i>	-2.8.21	<u>+2.5.7</u>
8	B9-18	4	18^2	<i>0.8066</i>	3p	3	3-girth	6	<i>Selfcomplem.</i>	-2.4.4	<u>+2.3.3</u>
9	B10-40	8	$5^1 40^1$	<i>0.9084</i>	5p	2	5-clique	32	<i>2-quinta-clique</i>	<u>-2.10.40</u>	<u>+2.8.25</u>
10	B10-15	3	$15^1 30^1$	<i>0.8328</i>	1c	10	5-girth	12	<i>Petersen gr.</i>	-2.3.2	<u>+4.10.15</u>
11	B10-30	6			1c	10	4-clique	5	<i>Petersen comp.</i>	-2.6.12	<u>+2.5.8</u>
12	B10-25	5	$20^1 25^1$	<i>0.8196</i>	2p	5	4-girth	-	<i>5-bi-clique</i>	-2.7.10	<u>+3.10.25</u>
13	B12-60	10	$6^1 60^1$	<i>0.9273</i>	6p	2	6-clique	64	<i>2-sexta-clique</i>	<u>-2.12.60</u>	<u>+2.10.41</u>
14	B12-54	9	$12^1 54^1$	<i>0.8868</i>	4p	3	4-clique	81	<i>3-quadro-clique</i>	-2.11.45	<u>+2.8.22</u>
15	B12-48	8	$18^1 48^1$	<i>0.8601</i>	3p	4	3-clique	64	<i>4-tri-clique</i>	-2.10.32	<u>+2.6.9</u>
16	B12-36	6	$30^1 36^1$	<i>0.8355</i>	2p	6	4-girth	-	<i>6-bi-clique</i>	-2.8.12	<u>+3.12.36</u>
17	B13-39	6	39^2	<i>0.8409</i>	1c	1	3-clique	22	<i>Selfcomplem.</i>	-2.5.7	<u>+2.4.5</u>
18	B14-84	12	$7^1 84^1$	<i>0.9399</i>	7p	2	7-clique	128	<i>2-septa-clique</i>	<u>-2.14.84</u>	<u>+2.12.61</u>
19	B14-49	7	$42^1 49^1$	<i>0.8470</i>	2p	7	4-girth	-	<i>7-bi-clique</i>	-2.9.14	<u>+3.14.49</u>
20	B15-90	12	$15^1 90^1$	<i>0.9119</i>	5p	3	5-clique	243	<i>3-quinta-clique</i>	-2.14.78	<u>+2.11.46</u>
21	B15-75	10	$30^1 75^1$	<i>0.8711</i>	3p	5	3-clique	125	<i>5-tri-clique</i>	-2.12.45	<u>+2.7.11</u>
22	B15-45	6	$45^1 60^1$	<i>0.8533</i>	1c	15	3-clique	-	<i>Weisfeiler</i>	-2.5.6	<u>+2.3.3</u>
23	B15-60	8			1c	15	5-clique	-	<i>Weisfeil. comp.</i>	-2.6.12	<u>+2.6.12</u>
24	B16-112	14	$8^1 112^1$	<i>0.9488</i>	8p	2	8-clique	256	<i>2-octa-clique</i>	<u>-2.16.112</u>	<u>+2.14.85</u>
25	B16-96	12	$24^1 96^1$	<i>0.8955</i>	4p	4	4-clique	256	<i>4-quadro-clique</i>	-2.14.72	<u>+2.10.33</u>
26	B16-40	5	$40^1 80^1$	<i>0.8670</i>	4p	4	4-girth	-	<i>Greenwood</i>	-2.4.4	<u>+3.10.13</u>
27	B16-80	10			1c	16	5-clique	16	<i>Greenw. comp.</i>	-2.8.24	<u>+2.8.22</u>
28	B16-48	6	$48^1 72^1$	<i>0.8594</i>	1c	16	4-clique	-	<i>Shrikhande</i>	-2.4.4	<u>+2.4.6</u>
29	B16-72	9			1c	16	4-clique	-	<i>Shrikhan comp.</i>	-2.8.18	<u>+2.6.11</u>
30	B16-64	8	$56^1 64^1$	<i>0.8557</i>	2p	8	4-girth	-	<i>8-bi-clique</i>	-2.10.10	<u>+3.16.64</u>
31	B17-68	8	68^2	<i>0.8589</i>	1c	17	3-clique	-	<i>Selfcomplem.</i>	-2.6.11	<u>+2.5.7</u>
32	B18-144	16	$9^1 144^1$	<i>0.9555</i>	9p	2	9-clique	512	<i>2-nona-clique</i>	<u>-2.18.144</u>	<u>+2.16.113</u>
33	B18-135	15	$18^1 135^1$	<i>0.9280</i>	6p	3	6-clique	729	<i>3-sexta-clique</i>	-2.17.120	<u>+2.14.79</u>
34	B18-108	12	$45^1 108^1$	<i>0.8796</i>	3p	6	3-clique	216	<i>6-tri-clique</i>	-2.14.60	<u>+2.8.13</u>
35	B18-81	9	$72^1 81^1$	<i>0.8626</i>	2p	9	4-girth	-	<i>9-bi-clique</i>	-2.11.18	<u>+3.18.81</u>
36	B20-180	18	$10^1 180^1$	<i>0.9607</i>	10p	2	10-clique	1036	<i>2-deca-clique</i>	<u>-2.20.180</u>	<u>+2.18.45</u>
37	B20-160	16	$30^1 160^1$	<i>0.9169</i>	5p	4	5-clique	1924	<i>4-quinta-clique</i>	-2.18.128	<u>+2.14.73</u>
38	B20-150	15	$40^1 150^1$	<i>0.9019</i>	4p		4-clique	625	<i>5-quadro-clique</i>	-2.17.105	<u>+2.12.46</u>
39	B20-100	10	$90^1 100^1$	<i>0.8682</i>	2p	10	4-girth	-	<i>10-bi-clique</i>	-2.12.20	<u>+3.20.100</u>

Explanations: **a)** The marking of structure show the numbers of vertices and edges. **b)** *deg* – degree. **c)** *SRV* – symmetry vector (Def. 3.1); **d)** *SR* – symmetry value (Prop. 3.4). **e)** *c* – number of components. **f)** *m* – number of parts. **g)** *n* – power of parts. **h)** *s* – number of cliques.

So it is recognized 39 bisymmetric-strongly regular structures with 4 to 20 vertices, mainly on the ground of disconnected partial cliques induced. The results of J. Petersen (B10-15), A. Titov (B13-39), B. Weisfeiler (B15-45), Greenwood-Gleason-Cleish (B16-40) in the realm of bisymmetry are random coincides, because the first be interested on valence-regularity, other on self-complementary, third on strong regularity, fourth on color-conjecture, others on isomorphism testing etc.

The lists of strongly regular graphs are incomplete. For example, in a special list [26] lacked 31 strongly regular graphs with to 20 vertices. A “most complete” list [27] where be given 33 structures, among these also *n-m-cliques* fail unfortunately **25** (B16-96), **29** (B16-72), **33** (B18-135), **34** (B18-108), **37** (B20-160) and **38** (B20-150).

In the “most complete” list of strongly regular graphs are showed all the to 20 vertices *bi-cliques*, as *complete bipartite graphs*, whereby bi-clique with 4 vertices called *square* and with 6 vertices called *unity*. There are also showed all the *2-m-cliques*, that have title *r-cocktail party graphs*, whereby with 6 vertices called *octahedral graph* and with 8 vertices *16-cell graph*. Other *n-m-cliques* called mostly *circular graphs*. There lack five *n-m-cliques* and the complement of a known strongly regular graph

It is touch with partial coincide the bisymmetry and strong regularity. Bisymmetry cover also disconnected structures and strong regularity can be exists also by mono-, poly- and partial symmetry. But the lasts no exist among the structures with to 20 vertices. Semiotic approach was fill the “white blotch” of

lists the strongly regular graphs, was pick out the essence of so far ignored clique regularity, and this that the complement of strongly regular graph is also strongly regular.

In such lists exists also many large graphs. For example, on a list [26] to find a graph with 999 vertices:

16	(999, 448, 172, 224)	-	-
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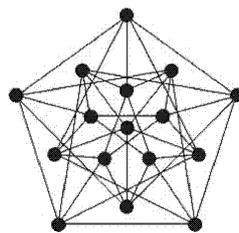
Example 2.10. We can in a simple way to induce some bisymmetric, clique- and strongly regular graphs with 999 vertices. In the lists of strongly regular graphs cannot these to find:

Nr	Notation	deg	E	SR	Regularity	Commentary	(+)signs
1	B999-2	2	999	0.9989	<i>3-clique</i>	333 disconnected partial 3-cliques	+2.3.3
2	B999-996	996	497502		<i>333-clique</i>	333 3-elementic parts 3-tricent-triginta-tri-clique	?
3	B999-8	8	3996	0.9979	<i>9-clique</i>	111 disconnected partial 9-cliques	+2.9.36
4	B999-990	990	494505		<i>111-clique</i>	111 9-elementic parts 9-cent-undeca-clique	?
5	B999-110	110	54945	0.9736	<i>111-clique</i>	9 disconnected partial 111-cliques	+2.111.6105
6	B999-888	888	443556		<i>9-clique</i>	9 111-elementic parts 111-nona-clique	?
7	B999-332	332	165832	0.9515	<i>333-clique</i>	3 disconnected partial 333-cliques	+2.333.55278
8	B999-666	666	332667		<i>3-clique</i>	3 333-elementic parts 333-tri-clique	?

Explanations: **a)** Strongly regular are there only $n-m$ -cliques. **b)** The names of $n-m$ -cliques can be for any no please, but others I cannot find.

With very important symmetry properties is graph **Gre** (B16-40) was constructed by Greenwood-Gleason as in any 3-colouring of the edges of the K_{16} without monochromatic triangles, the set of edges of each colour from this graph. It called also Clebish graph.

Example 2.11. Bisymmetric strongly regular Greenwood-Gleason-Clebish graph **Gre** and the structure models of **Gre** and its complement **GreC**:



$A: -2.4.4; B: +3.10.13.$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	i	AB	deg
0	B	-A	-A	B	-A	-A	B	-A	-A	-A	B	-A	-A	B	-A	1	105	5
	0	B	-A	-A	-A	B	-A	-A	B	-A	-A	-A	B	-A	-A	2	105	5
		0	B	-A	B	-A	-A	B	-A	-A	B	-A	-A	-A	-A	3	105	5
			0	B	-A	-A	4	105	5									
				0	B	-A	-A	-A	B	-A	-A	-A	B	-A	-A	5	105	5
					0	B	-A	-A	-A	-A	-A	-A	B	B	6	105	5	
						0	B	-A	-A	B	-A	-A	-A	-A	7	105	5	
							0	B	-A	-A	-A	-A	-A	B	8	105	5	
								0	B	-A	-A	B	-A	-A	9	105	5	
									0	B	-A	-A	-A	B	10	105	5	
										0	B	-A	-A	B	-A	11	105	5
											0	B	-A	-A	B	12	105	5
												0	B	-A	-A	13	105	5
													0	B	B	14	105	5
														0	-A	15	105	5
															0	16	105	5

A:-2.8.24; B:+2.8.22.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	i	A	B	deg
0	-A	B	B	-A	B	B	-A	B	B	B	-A	B	B	-A	B	1	510	10	
	0	-A	B	B	B	-A	B	B	-A	B	B	B	-A	B	B	2	510	10	
		0	-A	B	-A	B	B	-A	B	B	-A	B	B	B	B	3	510	10	
			0	-A	B	B	4	510	10										
				0	-A	B	B	B	B	-A	B	B	-A	B	B	5	510	10	
					0	-A	B	B	B	B	B	B	B	-A	-A	6	510	10	
						0	-A	B	B	-A	B	-A	B	B	B	7	510	10	
							0	-A	B	B	B	B	B	B	-A	8	510	10	
								0	-A	B	B	-A	B	-A	B	9	510	10	
									0	-A	B	B	B	B	-A	10	510	10	
										0	-A	B	B	-A	B	11	510	10	
											0	-A	B	B	-A	12	510	10	
												0	-A	B	B	13	510	10	
													0	-A	-A	14	510	10	
														0	B	15	510	10	
															0	16	510	10	

Common invariants and measures of graph and its complement:

Symmetry	V	R	K	N	SVV	SV	SRV	HR	SR
Bisymmetry	16	120	1	2	16 ¹	1.000	40 ¹ 80 ¹	0.2762	0.8670

Distinguishing invariants and measures:

G	E	k	N ^t	N ⁻	P	CL	MC	DM	SEV ^t	SE ^t	TRA	BRA
B16-40	40	1	1	1	2	2	4	2	40 ¹	1.000	0	0
B16-80	80	1	1	1	2	5	3	2	80 ¹	1.000	1.000	0

Explanations: a) The *bisymmetric* and *strongly regular* structure *Gre* is correspondingly to binary(+)sign +3.10.13 (a complete invariant!) *4-girth regular*, that mean *partiting*. This appear also to *4-partite* with incompletely connected parts on *4-elemental bases*. b) It is no quadroclique. c) The parts are *variety*, where, for example one variant is A=5,8,12,15; B=3,7,10,14; C=1,4,9,16; and D=2,6,11,13:

	A	B	C	D
A	0	4	6	10
B		0	10	6
C			0	4
D				0

d) From 4-elementic parts of *Gre* conclude the *4-clique regularity* of variety cliques of complement *GreC*. e) On the other hand, in case of each vertex of *Gre* its 5 adjacent vertices no have between themselves adjacencies (edges), from which conclude also a *5-clique-regularity* of complement *GreC*. We can in *GreC* to fix 16 different 5-cliques, such as (beginning at the adjacent vertices of first vertex of *Gre*) 2,5,8,12,15; 1,3,7,10,14; ... to ending with 6,8,10,12,14.

Conclusion 2.3. Semiotic approach discovers some new strongly- and clique regular structures.