Conclusion

The ten conditions of semiotic modelling of the graphs

<u>1. Structure</u> is the attribute of characterizing the ***relationships or organizing*** of elements of the discrete object. Structure ***GS*** is presentable as a ***graph G***. A *one-to-one correspondence* of graphs ***G*** that retain the structure ***GS*** is ***isomorphism.*** *Isomorphic graphs* have the same structure ***GS***, $\{G_1 \cong \ldots \cong G_q\} \rightarrow GS$.

About the structure can write the voluminous volumes, on the other hand, the meaning of structure is devalued to some kind of vague adjective. In fact, the structure is precisely definable and here presented is a concise summary of the many definitions. Associating the structure and graphs is for some unusual, but, for example, the structure of an institution or chemical compound should be clearly imagined.

<u>2. Semiotics.</u> Structure ***GS*** is something *qualitative* that only with the known graph-theoretical tools do not recognizable. The structure as such is investigated by help ***semiotics of structure***.

The meaning of the structure is mathematically undefined. They talk about the mathematical, algebraic and other structures, but the structure itself is not defined. On the structure, as such, are not interested and speak only about the specific problem- or object-oriented structures. For studying of the structure is necessary to "fall" or "rise up" to the level of semiotics.

<u>3. Binary relation.</u> It is confirmed that the recognition of structure reduces to the deep identification of binary relations r_{ij} between its elements with exactness up to *binary orbits* ΩR_n of vertex pairs of the group ***AutG***. The binary relations r_{ij} are identified with specific binary invariants, that we to ***binary signs*** call, among these the corresponding elements of products of the adjacency matrices E^n.

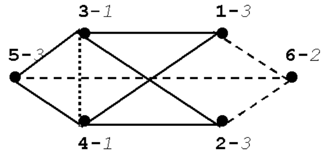
It is important initial condition, on which is based all the activity of structural research. The deep identification of binary relations can be seen as a “way of data mining”, or a “mode of structural arithmetic”. It can be for some like or not, but the important thing is that it works.

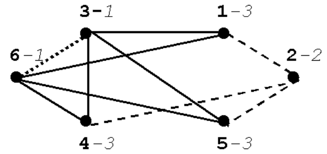
<u>4. Model of structure SM</u> is an ***ordered system of binary signs*** what recognizes the structure *with exactness up to binary orbits* and *isomorphism*.

<u>5. Position.</u> Structure model **SM** is *decomposed with exactness up to the orbits of elements* ΩV_k *and binaries* ΩR_n. ***Orbit*** Ω of ***AutG*** constitutes an *equivalence class*, which here should be called to ***position***.

The *orbits* of ***AutG*** and *positions* of the structure model **SM** coincide. The number of positions and their sizes determine the symmetry properties of the structure and allow classify these. The smaller number of positions, and the more their size, the structure is more symmetrical. Pairs of vertices belonging to a binary position ΩR_n form the ***position’s structure***.

<u>6. Structural equivalence $GS_A \approx GS_B$</u> and *isomorphism of graphs* $G_A \cong G_B$:





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C:+2.3.3; D:+2.5.7; E:+3.6.10.

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A:-2.5.7; B:-2.5.6;
C:+2.3.3; D:+2.5.7; E:+3.6.10.

1	1	2	3	3	3		u_i	k	s_i
3	6	2	1	4	5	i	ABCDE		123
0	D	-B	C	C	C	3	01310	1	103
	0	-B	C	C	C	6	01310	1	103
		0	E	E	E	2	02003	2	003
			0	-A	-A	1	20201	3	210
				0	-A	4	20201	3	210
					0	5	20201	3	210

Explanations:

a) Different graphs G_A and G_B have equivalent structure models $\mathbf{SM_A} \approx \mathbf{SM_B}$! This means that the graphs are ***isomorphic*** $G_A \cong G_B$ and structures are ***equivalent*** and the.

b) The element pairs are divided to ***five binary positions*** ΩR_n, wherein the adjacent elements or "edges" to *three binary(+)positions (full line, a dotted, dashed-line)* that coincides with binary signs ***C, D, E*** correspondingly. The structural elements are divided to ***three positions*** ΩV_k.

c) The column u_i of model consists of the ***frequency vectors***, which for the element ***i*** show its relations with other elements. On the basis of vectors u_i are arranged the positions ΩV_k in model.

d) The column s_i of model consists of the ***position vectors*** that represent the connections of element ***i*** with elements in corresponding positions ***k***. If on the framework of frequency vectors arises differences of position vectors, then by latest obtained a complementary partition into positions.

For recognition the equivalence of structural models A and B and isomorphism is sufficient to satisfy the three conditions: 1) *coincidence the sequences of binary signs* $\{\pm d.n.q._{ij}\}_A$ and $\{\pm d.n.q._{ij}\}_B$; 2) *coincidence of the frequency vectors* $\{u_i\}_A$ and $\{u_i\}_B$; 3) *coincidence the position vectors* $\{s_i\}_A$ ja $\{s_i\}_B$.

7. Adjacent structure. With *disjunctive removing* $\{G\backslash e_1 \vee G\backslash e_2 \vee \ldots \vee G\backslash e_q\}_n$, or *adding* $\{G\cup e_1 \vee G\cup e_2 \vee \ldots \vee G\cup e_q,\}_n$, a connection in the framework of a binary position ΩR_n obtained *largest sub-graphs* $\{G\backslash e_1 \cong G\backslash e_2 \cong \ldots \cong G\backslash e_q,\}_n$, or *smallest supergraphs* $\{G\cup e_1 \cong G\cup e_2 \cong \ldots \cong G\cup e_q,\}_n$, are *isomorphic* and constitute ***adjacent-substructures*** GS^{sub}_n or ***adjacent-superstructures*** GS^{sup}_n correspondingly.

It is an essential lawfulness that already in 1973 had published a modest but has been ignored for decades. The reason for this is probably that *orbit*, i.e. the *position* in structural terms, is considered to be so specific attribute of the group theory, that them are few who deign to link these to the graphs. Binary positions as such no one has previously observed.

8 Morphism. Disjunctive operation $F_n = \{(e_{ij})_1 \vee \ldots \vee (e_{ij})_q\}_n$ in the framework of binary position ΩR_n that changes the structure ***GS*** to its adjacent structure GS^{adj}_n, called ***morphism*** F_n, $F_n: GS \rightarrow GS^{adj}_n$. Morphism is ***reversible*** F^{rev}_n: each adjacent structure GS^{adj}_n has a reverse position ΩR^{rev}_n, where corresponding morphism reconstruct F^{rev}_n (restore) the initial structure, $F^{rev}_n: GS^{adj}_n \rightarrow GS$.

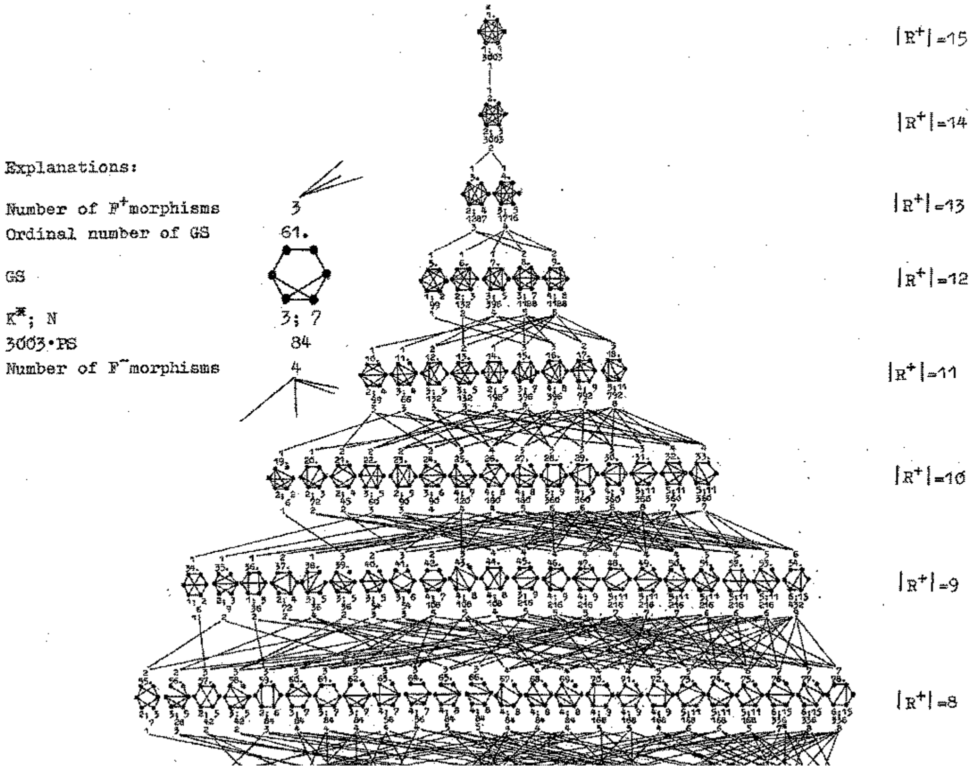
The existence of a morphism ensues directly from the existence of adjacent structures. Binary positions ΩR_n of the structure ***GS*** are on the aspect of its adjacent structures GS^{adj}_n all reverse positions ΩR^{rev}_n.

9. Factorability (decomposability) and reconstructing (restorability). If morphisms $F_n: GS \rightarrow GS^{adj}_n$ disjunctively $F_1 \vee \ldots \vee F_n \vee \ldots \vee F_N$ are applied to the binary orbits $\Omega R_1, \ldots, \Omega R_n, \ldots, \Omega R_N$, of the structure ***GS***, then is structure ***GS factorized (decomposed)*** to its *adjacent structures* $GS^{adj}_1, \ldots, GS^{adj}_n, \ldots, GS^{adj}_N$. The reversibility of the morphism ensures the ***reconstructing*** on the base of its adjacent structures (which does not mean that obligatory on the same binary operations).

About the problem of recoverability matters should emphasize that it takes place on the base of reverse binary position ΩR^{rev}_n, i.e. on the base of an *arbitrary* operation in this position. For many decades the attempts of proving the restoration on the base the wording of Ulam's Conjecture were unsuccessful and it is proved only for some types of graphs – there not general solution. The considering of recoverability on the level of isomorphism classes or structure makes its proving on the ground of wording the Ulam's Conjecture to a meaningless hobby. The proving of recoverability on the level of isomorphism classes has recommended by grandmaster W. T. Tutte yet in 1998. ***Restorability of structure is a reverse operation of factorability*** – whole story! I would hope that my for this conception not accuse in heresy.

10. Sequence and system of adjacent-structures. *Sequence of adjacent structures* ***SF*** changes a structure to its some kind sub- or superstructure: $SF = {}^{F1:}GS_0 \rightarrow {}^{F2:}GS_1 \rightarrow {}^{F3:}GS_2 \rightarrow \ldots {}^{Ft:}GS_{t-1} \rightarrow GS_t$. Structural changes may be take place in the form *assemblages of sequences*. Assemblage of sequences of adjacent structures between an empty structure GS° and complete structure $GS^{\bullet}$ constitutes ***the system of all the structures with n elements***.

The upper side of ***lattice*** of the *system of all the structures with six elements* $\mathfrak{G}^{|V|=6}$:



Explanations:

- **a)** $|\mathbf{R}^+|$ denote the ***structural level m,*** i.e. the number of connections (i.e. "edges") in the structures.
- **b)** Each graph presents there its *isomorphism class* or ***structure GS***, each link presents ***morphism F***.
- **c)** Each structure ***GS*** in this lattice is an ***adjacent structure*** $\boldsymbol{GS^{adj}_n}$ of some other structures.
- **d)** The ***complements*** of structures placed symmetrically in the lower side of lattice.
- **e)** Essential meanings in the systems $\mathfrak{G}^{|V|}$ have ***probability characteristics*** of morphisms ***PF*** and on this base obtained ***existence probabilities PS.***
- **f)** The number of structures with six elements is 156, the number of morphisms is 572.

Summary:

- The structure is a *complete invariant of isomorphic graphs*.
- With deep-identification of binary relations obtained structure model recognizes the structure *with exactness up to binary positions and isomorphism*.
- The *factorability (decomposability)* of the structure on the base of its binary positions to adjacent structures and *reconstructing (restorability)* on the base of adjacent structures are *equivalent but opposite operations.*
- The assemblages of successions of adjacent structures *form correct systems of structures with n-elements*.

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