4. STRUCTURAL TRANSFORMATIONS

Structural transformations take place in case *transition* from a graph ***G*** to another graph ***H***. *Elementary structural transformation* is a transition from graph ***G*** to its subgraph $G\backslash v_i$ or $G\backslash e_{ij}$. To the relationships between graphs and its subgraphs was interested beginning at the formulation of Ulam Conjecture [44].

Suitable to remind a theorem about the relationships between ***G*** and its subgraphs $G\backslash v_i$, proved by V. Titov in 1975 [41], which at that time, unfortunately, has not found the attention.

Titov's theorem. If all the ($G\backslash v_i$)-sub-graphs of graph ***G*** are isomorphic, then automorphism group *Aut**G*** is transitive on the set of vertices ***V.***

It mean that graph ***G*** is *transitive* or *vertex symmetric,* i.e. there exists only one vertex position $\Omega V_{k=1=K} = \Omega(v_{i=1},\dots,v_{i=|V|})_{k=1=K}$, which correspond just to one isomorphism class of ($G\backslash v_i$)-sub-graphs, $\Gamma_{k=1=K} = (G\backslash v_{i=1} \cong \dots \cong G\backslash v_{i=|V|})_{k=1=K}$.

4.1. Structural transformations and reconstructions

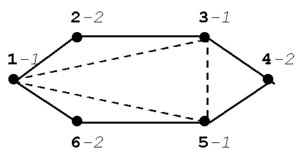
By removing an edge $G\backslash e_{ij}$ of ***G*** obtained a *greatest subgraph* G^{sub}**.** The number of G^{sub} equals to the number of edges. With adding an edge $G\cup e_{ij}$ to ***G*** obtained a *smallest supergraph* G^{sup}. The number of G^{sup} equals to the number of "non-edges".

Definition 4.1. Greatest subgraphs G^{sub} and smallest supergraphs G^{sup} called ***adjacent graphs*** G^{adj} of ***G.***

Proposition 4.1. If the adjacent graphs G^{adj} are obtained on the ground of the same binary position ΩR_n then are these ***isomorphic*** and constitute an ***adjacent structure*** GS^{adj}_n of ***GS***. The number of adjacent structure equals to the number of binary positions.

Corollary 4.1. Disjunctive edge operation $F_n=\{(f_{ij})_1 \vee\dots\vee (f_{ij})_q\}_n$ in the frame of a binary position ΩR_n that ***transforms*** the structure ***GS*** to its adjacent structure GS^{adj}_n is called ***morphism***, $F_n: GS\rightarrow GS^{adj}_n$.

Example 4.1. ***Partially symmetric*** structure *GS.37(6.9.4)* (see chapter 5) with two element positions and four binary positions, its graph, structure model, characteristics of changes and morphisms:




```
A:-2.4.5; B:-2.3.2;
C:+2.3.3; D:+2.4.5.

| 1  1  1| 2  2  2|       ui     k  si
| 1  3  5| 2  4  6|  i    ABCD       12
|  0  D  D| C -A  C|  1    1022   1  22
      0  D| C  C -A|  3    1022   1  22
         0|-A  C  C|  5    1022   1  22
          | 0 -B -B|  2    1220   2  20
               0 -B|  4    1220   2  20
                  0|  6    1220   2  20
```

	GS^{adj}_n	1	2
GS.37	GS^{sup}_{n-}	29	30
	k.k' (p)	2.2 (-B)	1.2 (-A)
	PF^{sup}_{n-}	3/6	3/6
GS.37	GS^{sub}_{n+}	72	76
	k.k' (p)	1.1 (+D)	1.2 (+C)
	PF^{sub}_{n+}	3/9	6/9

Explanations:

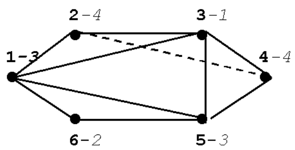
a) GS^{sup}_{n-} and GS^{sub}_{n+} denotes the *ordering numbers* of adjacent superstructures and adjacent substructures in the system of structures with six elements (example 4.4);

b) ***k,k'*** – index of partial model $SM_{k,k'}$, whither belong the binary position ***(p)***;

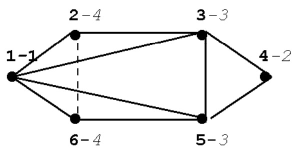
c) PF_n – *morphism probability*.

Example 4.2. Three *isomorphic graphs* that represent the ***adjacent superstructure*** $GS^{sup}_{n=-B}$, (*GS.29)* (chapter 5) of structure *GS.37* (example 4.1). These are obtained by *adding* the connections 2-4 or 2-6 or 4-6 (dashed line) to binary(–)position *–B* of *GS.37.* Their *common binary signs* and *equivalent models* $\mathbf{SM_1 \equiv SM_2 \equiv SM_3}$:

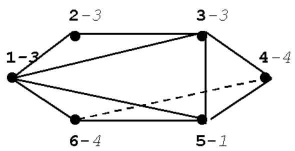
```
A:-2.5.8; B:-2.4.5; C;-2.3.2;
      D:+2.3.3; E:+2.4.5.
```




```
| 1| 2| 3   3| 4   4|       ui          si
| 3| 6| 1   5| 2   4|   i  ABCDE  k  1234
| 0|-B| E   E| E   E|   3  01004  1  0022
   | 0| D   D|-C  -C|   6  01220  2  0020
      | 0   E| D  -A|   1  10022  3  1111
            0|-A   D|   5  10022  3  1111
             | 0  D*|   2  10121  4  1011
                  0|    4  10121  4  1011
```




```
| 1| 2| 3   3| 4   4|       ui          si
| 1| 4| 3   5| 2   6|   i  ABCDE  k  1234
| 0|-B| E   E| E   E|   1  01004  1  0022
   | 0| D   D|-C  -C|   4  01220  2  0020
      | 0   E| D  -A|   3  10022  3  1111
            0|-A   D|   5  10022  3  1111
             | 0  D*|   2  10121  4  1011
                  0|    6  10121  4  1011
```

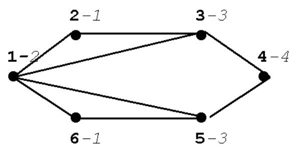



```
| 1| 2| 3   3| 4   4|       ui          si
| 5| 2| 1   3| 4   6|   i  ABCDE  k  1234
| 0|-B| E   E| E   E|   5  01004  1  0022
   | 0| D   D|-C  -C|   2  01220  2  0020
      | 0   E|-A   D|   1  10022  3  1111
            0| D  -A|   3  10022  3  1111
             | 0  D*|   4  10121  4  1011
                  0|    6  10121  4  1011
```

Explanation: The equivalent structure models differ from each other only by numbering of elements in the positions.

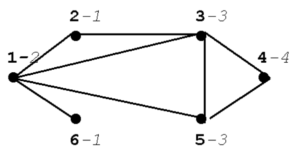
Example 4.3. The *different* ***adjacent substructures*** $GS^{sub}_{n=+D}$, (*GS.72)* (chapter 5) and $GS^{sub}_{n=+C}$, (*GS.76*) of structure *GS.37* (example 4.1) that obtained by *removing* the connection 3-5 from binary(+)position ***+D*** and *removing* the connection 5-6 from binary(+)position ***+C*** correspondingly. Their *non-isomorphic graphs, different binary signs* and *non-equivalent structure models* $\mathbf{SM}_A$ and $\mathbf{SM}_B$:

```
A:-2.4.4; B:-2.3.2;
C:+2.3.3; D:+3.4.4.
```




```
| 1   1| 2| 3   3| 4|       ui         si
| 2   6| 1| 3   5| 4|   i  ABCD  k  1234
| 0  -B| C| C  -B|-B|   2  0320  1  0110
      0| C|-B   C|-B|   6  0320  1  0110
         | 0| C   C|-A|   1  1040  2  2020
            | 0-A*| D|   3  1121  3  1101
                 0| D|   5  1121  3  1101
                    0|   4  1202  4  0020
```

```
A:-3.5.6; B:-2.4.5; C:-2.3.2;
D:+1.2.1; E:+2.3.3; F:+2.4.5.
```




```
| 1| 2| 3| 4| 5| 6|        ui            si
| 3| 1| 5| 2| 6| 4|   i ABCDEF  k  123456
| 0| F| F| E|-C| E|   3 001022  1  011101
      0| E| E| D|-B|   1 010121  2  101110
         | 0|-B|-C| E|   5 011021  3  110001
            | 0|C*|-C|   2 012020  4  110000
                0|-A|   6 103100  5  010000
                   0|   4 111020  6  101000
```

Explanation: On the basis of various binary positions obtained adjacent structures are *not equivalent*.

Each structure ***GS*** is an adjacent substructure GS^{sub}_n or adjacent superstructure G^{sup}_n of some other structures. For each binary position ΩR_n correspond an adjacent structure GS^{adj}_n.

Proposition 4.2. If morphisms F_n: $GS \rightarrow GS^{adj}_n$ are applied to binary positions $\Omega R_1,\ldots,\Omega R_n,\ldots,\Omega R_N$ of ***GS*** disjunctively, $F_1 \vee\ldots\vee F_n \vee\ldots\vee F_N$, then ***GS*** is ***transformed*** to its *adjacent structures* $GS^{adj}_1,\ldots,GS^{adj}_n,\ldots,GS^{adj}_N$.

Corollary 4.2. ***Not-transformable structures do not exist.***

Proposition 4.3. Morphism ***F*** is ***inverses –*** in each adjacent structure GS^{adj} of ***GS*** exist an "*inverse position*" ΩR^{inv}, whereat used *inverse morphism* F^{inv} ***reconstruct*** the initial structure ***GS***, F^{inv}: $GS^{adj} \rightarrow GS$.

Let the structure on example 4.2 is an initial structure ***GS*** that has an adjacent substructure GS^{sub}_n in the forms of structure on example 4.1. Then ***GS*** can be *reconstruct* by adding a connection to the *reverse position –B* of GS^{sub}_n with morphism probability PF^{inv}=3/6.

Propositions 4.4. The relations between transformations and reconstructions of structure:

P4.4.1. If structure ***GS*** is ***transformed*** to its ***adjacent substructures*** $GS^{sub}_1,\dots,GS^{sub}_n,\dots,GS^{sub}_N$, then ***GS*** is the *common adjacent superstructure* GS^{sup} of all its adjacent substructures GS^{sub}_n and is ***reconstructable*** by an *inverse morphism* F^{inv}_n: $GS^{sub}_n \rightarrow GS$ to each its adjacent substructure.

P4.4.2. If structure ***GS*** is ***transformed*** to its ***adjacent superstructures*** $GS^{sup}_1,\dots,GS^{sup}_n,\dots,GS^{sup}_N$, then ***GS*** is the *common adjacent substructure* GS^{sub} of all its adjacent superstructures GS^{sup}_n and is ***reconstructable*** by an *inverse morphism* F^{inv}_n: $GS^{sup}_n \rightarrow GS$ to each its adjacent superstructure.

Thus, structure ***GS*** is ***reconstructable*** by its *adjacent substructures* GS^{sub}_n and its *adjacent superstructures* GS^{sup}_n. This coexistence makes the reconstruction to an ***inverse transformation***.

Corollary 4.3. ***Not-reconstructive structures do not exist***.

The *reconstruction problem* is known as ***Ulam's Conjecture*** that reflects the isomorphism relations between two graphs and their ($G \backslash v_i$)-subgraphs [44]. It is formulated as follows: "If for each ***i***, the sub-graphs $G_i = G \backslash v_i$ and $H_i = H \backslash u_i$ are isomorphic, then the graphs ***G*** and ***H*** are isomorphic".

This problem has been over the past half century, one of under active consideration graph theoretical problem, but the ultimate solutions have only some graph classes. Why so? On the structural aspect are the attempts of solution the conjecture by its wording *senseless,* because, if given graphs ***G*** and ***H*** then on the ground of *structure models* SM_G and SM_H we obtain the complete information about corresponding graphs, their isomorphism or non-isomorphism and of their adjacent graphs. Other approaches are meaningless for us here.

Evidently be interested on the question**:** contains the collection of subgraphs $G \backslash v_i$ of ***G*** enough information about graph ***G*** itself? Ulam's Conjecture treats the reconstruction on the aspect of removing of the vertices, but we treat it on the aspect of adding and removing of edges. This not changes the essence of reconstruction, because all remains to the frame of graphs (structures) and their adjacent-graphs (adjacent-structures), i.e. in our case to the frame of *morphisms* F_n.

Already old master W. T. Tutte emphasized that reconstruction-problem must be solve on the basis of *isomorphism classes, (i.e. structures)* that we also have followed [43].

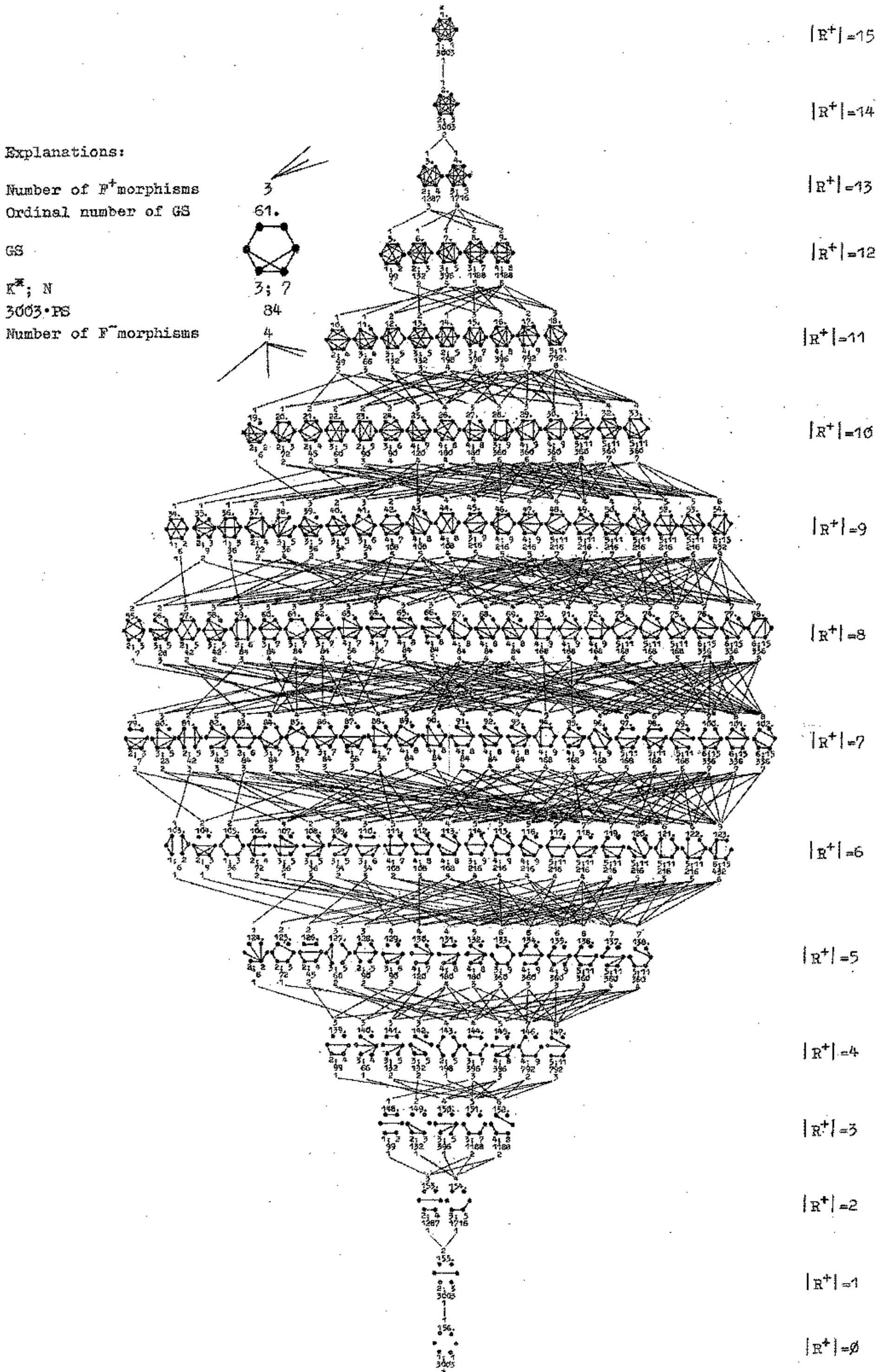
4.2. Systems of structural transformations

Each set of all the non-isomorphic graphs with ***n*** vertices constitute a *system of adjacent structures* i.e. *system of structural transformations.* Each structure can to its adjacent structures transformed and each structure is an adjacent structure of some structures. Formed many collections of graphs with ***n*** vertices, but the adjacencies do not notice. Why is that?

The first sample of non-isomorphic graphs with up to six vertices was represented by Frank Harary in 1969th [10]. Later, F. Harary and E. Palmer had calculated the number of non-isomorphic graphs (i.e. structures) up to 24 vertices [11]. R. Read and F. Wilson have given the diagrams of graphs also up to seven vertices [24]. But so far about the relationships between structures, i.e. morphisms, do not discussed – their do not wanted to notice.

By help of morphisms F_n are generated the *system of adjacent structures with five elements* [36] (where 72 morphisms connect 34 structures) and the *system with six elements* [37] (where 572 morphisms connect 156 structures). It can be generated for all the structures and shows the inevitability of reconstructing.

<u>Example 4.4.</u> The ***lattice of transformations of the structures*** *with six elements* (see also Supplement):



Explanations:

- **a)** $|\mathbf{R}^{+}|$ denote the *structural level* ***m****,* i.e. the number of connections (i.e. "edges") in the structures.
- **b)** Each graph presents there its *isomorphism class* or *structure* ***GS***.
- **c)** Each structure in this lattice is an *adjacent structure* of some other structures, where the edges represent the morphisms F_n.
- **d)** The *complements* of represented structures placed symmetrically in the upper and lower half of lattice.

Generating of the adjacent structures proceed by the morphisms $\boldsymbol{F_n}$, so that in the framework of structures ***GS*** of a structural level ***m*** (i.e. structures with a concrete number of "edges") be formed the structure models of adjacent structures $\boldsymbol{GS^{adj}}$. And so proceed, from a level to its adjacent level [33]. In result is obtained the *system of structural transformations* or *system of adjacent structures* $\mathfrak{G}$. Generating can be begin from zero or complete structure (see also Supplement).

Example 4.5. The number of graphs in the samples of non-isomorphic graphs with 3 to 10 vertices**:**

Number of elements $\lvert V\rvert$	Number of structures ***p***	Among this connected ***p****	Number of levels ***m***
3	4	2	4
4	11	6	7
5	34	21	11
6	156	112	16
7	1044	853	22
8	12346	11117	29
9	274668	261080	37
10	12005168	11716571	46

Propositions 4.5. Some ***general properties*** of the systems $\mathfrak{G}^{\lvert V\rvert}$:

P4.5.1. If the number of structural levels ***m*** in system $\mathfrak{G}^{\lvert V\rvert}$ is *even number* (as in case $\lvert V\rvert=6$ and $\lvert V\rvert=7$), then it lattice is *bilaterally symmetric* with regard its bisector, which separates the *structures* ***GS*** from their *complements* $\urcorner GS$.

P4.5.2. If the number of structural levels ***m*** in system $\mathfrak{G}^{\lvert V\rvert}$ is *odd number* (as in case $\lvert V\rvert=4$, $\lvert V\rvert=5$, $\lvert V\rvert=8$ and $\lvert V\rvert=9$), then the bisector is a structural level in which be located the *structures* ***GS***, their *complements* $\urcorner GS$ and also *self-complemented* structures $GS=\urcorner GS$.

Essential meaning have the probabilistic characteristics.

Propositions 4.6. ***Probabilistic characteristics*** of the systems $\mathfrak{G}^{\lvert V\rvert}$:

P4.6.1. *Randomness* in the systems $\mathfrak{G}$ based on the *morphism probabilities* $\boldsymbol{PF_n}$.

P4.6.2. There exists *transition probability* $\boldsymbol{P_{ij}}$ at a structure $\boldsymbol{GS_i}$ to a non-adjacent structure $\boldsymbol{GS_j}$.

P4.6.3. Transition probabilities $\boldsymbol{P_{ij}}$ form the *stationary Markov chain* ***PM*** of structural genesis.

P4.6.4. *Existence probability* ***PS*** of structure ***GS*** in the structural level $\lvert\mathbf{R}^+\rvert$ of system $\mathfrak{G}$ is expressed in the form:

$$PS={}_{n=1}\sum^{N-}PS^{sup}{}_{n}\times PF^{sub}{}_{n}={}_{n=1}\sum^{N+}PS^{sub}{}_{n-}\times PF^{supp}{}_{n},$$

where ***n*** is the structural index of binary position, $\boldsymbol{PS^{sup}{}_n}$ existence probability of adjacent superstructure and $\boldsymbol{PF^{sub}{}_n}$ its morphism probability.

P4.6.5. The *sum of existence probabilities* ***PS*** of structures in the structural level $\lvert\mathbf{R}^+\rvert$ equal to one, $\boldsymbol{\sum PS=1}$.

P4.6.6. Existence probabilities of *structure* and its *complement* are equal, $PS(GS)=PS(\urcorner GS)$.

P4.6.7. Existence probabilities ***PS*** are *rational numbers* and are directly related with the degree of genesis $\lvert V\rvert$.

P4.6.8. Distribution of the probabilities ***PS*** in the structure levels approach to *logarithmic normal distribution.*

Since morphisms have by structural transformations principal role, they are suitable to represent any propositions about them. Some algebraic systems can express some fragments or aspects of structural transformations and can have corresponding models.

Proposition 4.7. The class of morphisms **F** forms an ***additive group A*** from the aspect of the compositions ***F&F*** in the system $\mathfrak{G}^{\lvert V\rvert}$.

Let us check the validity of additive group postulates for morphisms:

1) $F_a\&(F_b\&F_c)=(F_a\&F_b)\&F_c$ (distributivity).

2) $\forall(F_a,F_b);\ F_a,F_b\in A,\ \exists F_c,\ F_a\&F_b=F_c\in A$ (because composition of morphisms is also a morphism).
3) $\forall F\ \exists!F';\ F,F'\in A,\ \forall(F,F')\ \exists F\&F'=F^0\in A$ (existence of an opposite morphism).
4) $\exists!F^0;\ F^0\in A,\ \forall(F,F^0)=F\in A$ (existence of a zero morphism).

The postulates of the *category* C satisfy the system $\mathfrak{G}^{|V|}$ most. S.Eilenberg and S.MacLane [7] formulated the foundations of category C in 50th years in the framework of elaboration of algebraic methods for topology. The notion "morphism" is a principal concept of category C.

Proposition 4.8. The class of G-structures $\{GS^{|V|}\}$ together with morphism's class **F** of the system $\mathfrak{G}^{|V|}$ forms a ***category C.***

There exist correspondences of postulates of category with attributes of the system $\mathfrak{G}^{|V|}$. Let the class $\{GS^{|V|}\}$ correspond to object class Ob of C.

1) $\forall(GS_i,\ GS_j)\in\{GS^{|V|}\},\ \exists\{F\}:\ (GS_i\to GS_j)$, that represents a set of possible morphisms from GS_i to GS_j, denoted by $\{F\}\subset\mathrm{Hom}(GS_i,GS_j)$, which can be constitute various successions of structures GS.
2) For each $(GS_i,GS_j,GS_k)\in\{GS^{|V|}\}$ exist mapping $\mathrm{Hom}(GS_i,GS_j)\&\mathrm{Hom}(GS_j,GS_k)\to\to\mathrm{Hom}(GS_i,GS_k)$, since $F_a\in\mathrm{Hom}(GS_i,GS_j)$ and $F_b\in\mathrm{Hom}(GS_j,GS_k)$, where the result of composition of morphisms (or succession, as an high degree morphism) $F_a\&F_b=F_c\in\mathrm{Hom}(GS_i,GS_k)$.
3) Morphisms $\mathrm{Hom}(GS_i,GS_j)$ and their compositions satisfy the category postulates, because:

- In the case of each succession $^{Fa:}GS_i\to^{Fb:}GS_j\to^{Fc:}GS_k\to$ the associability $F_a\&(F_b\&F_c)=(F_a\&F_b)\&F_c$ be valid;
- $\forall GS\in\{GS^{|V|}\}\ \exists F^0=F\&F';$ where $F^0:\ GS\to GS$ constitute identity morphism or an unit of GS;
- If the pairs (GS_i,GS_j) and (GS_i',GS_j') are different, then $\mathrm{Hom}(GS_i,GS_j)\cap\mathrm{Hom}(GS_i',GS_j')=\varnothing$ which mean the existence of disjoint connecting graphs $\mathfrak{G}^{|V|}_{ij}\cap\mathfrak{G}^{|V|}_{ij}{}'=\varnothing$ in the system $\mathfrak{G}^{|V|}$.

In structural genesis has important role *randomness.* This is expressed in the form of *selection the adjacent structures*, i.e. elementary structural changes. The *probabilistic characteristics* are related with *internal diversity* of structure, i.e. binary positions, and have essential meaning in structural research.

Probabilistic characteristics exist also for successions of structures etc.

4.3. Successions of structural transformations

The interest for succession of structures, i. e. *structural transformations* is connected not only with the study of their lawfulness. The structural transformations have also been essential in cognitive and applicative analysis, research and simulation of phenomena and processes where the existence can be expressed as the gradual changes of structure.

Successive elementary transformation of structures GS can be expressed as the paths in the lattice of system $\mathfrak{G}^{|V|}$ or can be formed independently.

Definition 4.2. An ordered set of morphisms $F_1\&F_2\&\ldots\&F_t$ to structures GS,

$$^{F1:}GS_0\to{}^{F2:}GS_1\to{}^{F3:}GS_2\to\ldots{}^{Ft:}GS_{t-1}\to GS_t,$$

is a ***succession of morphisms,*** denoted by ***SF.***

Propositions 4.9. The properties of *successions* ***SF*****:**

P4.9.1. A succession ***SF*** can proceed ***randomly*** or ***non-randomly***. If the selection of morphisms takes place on the ground of certain conditions or criterions, then it is a ***teleological succession.***

P4.9.2. The successions between non-adjacent structures GS_i and GS_j, in the lattice of the system $\mathfrak{G}^{|V|}$ constitutes an ***assemblage of successions***.

P4.9.3. Structural changes that take place only by F^+morphisms or only by F^-morphisms form a ***vertical succession.***

P4.9.4. A succession ***SF*** whose initial structure GS_i and result structure GS_j can be found on the same subsystem $\mathfrak{G}^m$ form a ***horizontal succession.*** Such structural changes are based on the morphism's pair $F^-\&F^+$ (or $F^+\&F^-$), which constitutes a *"displacement of connection"* in structure ***GS***.

P4.9.5. A succession ***SF*** where the structural values change monotonously, constitutes ***monotonous succession SFM*** on the sense of corresponding characteristics.

P4.9.6. A succession ***SF*** where the values of a structural characteristic stay unchangeable or change not much, constitutes a ***stable succession SFS***. A set of such successions (in the form of a subsystem $\mathfrak{G}^{|V|}{}_{ij}$) constitutes a ***stability assemblage SA*** in the sense of certain structural characteristics. A compact set of structures with equal structural values in the system $\mathfrak{G}^{|V|}$, which do not constitute stability assemblage, form a ***stability domain SD.***

P4.9.7. ***Probability of random succession PSF*** with length ***t*** constitutes the product of corresponding morphism probabilities,

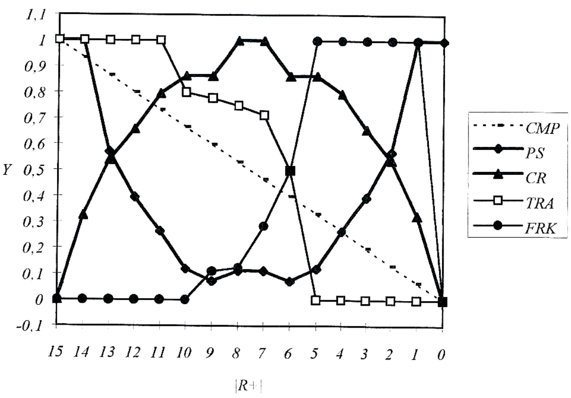
$$PSF={}_{i=1}\prod^{t}PF_i=PF_1\times PF_2\times\ldots\times PF_t.$$

Structure constitutes something *qualitative.* Thus, each possible structural transformation is a *change of quality*. At the same time is structure *measurable* from various aspects. Each separate measure, i.e. structural characteristic *H*, expresses evidently a single quantitative aspect, but with a certain set of structural characteristics it is possible to recognize also qualitative properties of the structure.

Now we can speak only on differences on the aspect of their structural characteristics. Does such ***vertical succession***, which pervade all the structural levels so, that its ***structural characteristics change monotonously?*** It is ascertained, that for the great number of various vertical successions in $\mathfrak{G}^{|V|=6}$ there exists only one single succession, which satisfies such strict conditions completely $GS_1\rightarrow GS_2\rightarrow GS_4\rightarrow GS_8\rightarrow GS_{17}\rightarrow GS_{33}\rightarrow GS_{53}\rightarrow GS_{76}\rightarrow GS_{100}\rightarrow GS_{122}\rightarrow GS_{138}\rightarrow GS_{146}\rightarrow GS_{151}\rightarrow GS_{154}\rightarrow GS_{155}\rightarrow GS_{156}$. Such structural characteristics are compactness *CMP*, existence probability *PS*, symmetry value *SR* (or contrary –asymmetricality *CR*), triangularity *TRA*, branching *FRK*, complexity *CPX*, topological entropy *HE*, diameter *DMR*, cliqueability *MCQ* and information capacity *HV.*

Such monotonous succession and corresponding structural characteristics was useable by constructing of an elegant but very abstract ontogeny phenomenon – at origin to ripeness [29].

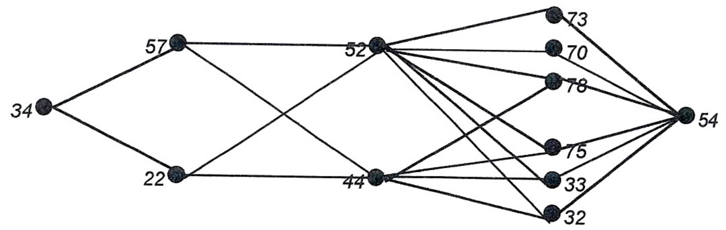
Example 4.6. A diagram of the *monotonous succession* ***SFM*** in the system $\mathfrak{G}^{|V|=6}$**:**



Explanation: a) *Y* – rationed value of structural characteristics; *R* – ordering number of structural level ***GSL.*** b) If the monotonous succession ***SFM*** is random then its probability $PSF_B^-=2/(4725\times3003)=1.41\times10^{-7}$ is of extremely little value.

In $\mathfrak{G}^{|V|=6}$ can be exist ***the assemblages of horizontal successions*** between two structurally extremely different ***GS.*** For example, in the subsystem $\mathfrak{G}^{m=9}$ between the symmetrical structure GS_{34} *(SR=0.751)* and totally **a**symmetric structure GS_{54} *(SR=0).* Let us fix the partial system $\mathfrak{G}_{34;54}$ as an assemblage. The horizontal successions include the structures ***GS*** from neighbouring subsystems $\mathfrak{G}^{m=8}$ and $\mathfrak{G}^{m=10}$ and are founded on $F^{-}\&F^{+}$ morphisms. The length of successions ***d****=4* (with only two "displacements"), the number of structures in $SAS_{34;54}$ ***n****=12* and the number of morphisms ***m****=22.* Single successions of the assemblage, for example $GS_{34}\rightarrow GS_{57}\rightarrow GS_{52}\rightarrow GS_{78}\rightarrow GS_{54}$, are mainly monotonous. This example is essential for the study of random aspects of structural changes.

Example 4.7. Partial system $\mathfrak{G}^{|V|=6}_{34,54}$ representing an *assemblage of horizontal successions*:



SR=0.751 *SR=0.461* *SR=0.273* *SR=0.137* *SR=0*

Explanation: Symmetry values are represented by the lower monotonous succession.

The probabilities of random successions in assemblage of horizontal successions in subsystem $\mathfrak{G}^{|V|=6}_{34,54}$ are products of random morphisms. Probability of changing a symmetric structure GS_{34} to a completely **a**symmetric structure GS_{54} is $PSF_{C1}=9/15\times4/15\times2/15\times1/15=72/50625$. The same succession, but in contrary direction – changes completely **a**symmetric structure GS_{54} to a symmetric structure GS_{34} with probability $PSF_{C2}=1/15^4=1/50625$. Thus probability of changing an **a**symmetric structure ***GS*** to a symmetric ***is 72 times less*** than in opposite case! It is rather a rule than a chance.

Example 4.8. *Stationary Markov chain* $PM_{34,54}$ for assemblage of horizontal successions in subsystem $\mathfrak{G}^{|V|=6}_{34,54}$:

GS	34	22	57	44	52	32	33	75	78	70	73	54
34	*0*	*1 20250*	*1 30375*	*2 8100*	*2 16200*	*3 2160*	*3 2160*	*3 2160*	*3 4320*	*3 2160*	*3 2160*	*4 1728*
22	*1 3375*	*0*	*2 4050*	*1 10125*	*2 20250*	*2 2700*	*2 2700*	*2 2700*	*2 5400*	*2 2700*	*2 2700*	*3 2160*
57	*1 3375*	*2 2700*	*0*	*1 6750*	*1 13500*	*2 1800*	*2 1800*	*2 1800*	*2 3600*	*2 1800*	*2 1800*	*3 1440*
44	*2 450*	*1 3375*	*1 3375*	*0*	*2 4725*	*1 6750*	*1 6750*	*1 6750*	*1 13500*	*3 405*	*3 405*	*2 3600*
52	*2 350*	*1 3375*	*1 3375*	*2 1800*	*0*	*1 3375*	*1 3375*	*1 3375*	*1 6750*	*1 6750*	*1 6750*	*2 3600*
32	*3 60*	*2 450*	*2 450*	*1 3375*	*1 3375*	*0*	*2 1125*	*2 1125*	*2 2025*	*2 2700*	*2 2700*	*1 6750*
33	*3 60*	*2 450*	*2 450*	*1 3375*	*1 3375*	*2 1125*	*0*	*2 1125*	*2 2025*	*2 2700*	*2 2700*	*1 6750*
75	*3 60*	*2 450*	*2 450*	*1 3375*	*1 3375*	*2 1125*	*2 1125*	*0*	*2 2025*	*2 2700*	*2 2700*	*1 6750*
78	*3 60*	*2 450*	*2 450*	*1 3375*	*1 3375*	*2 1125*	*2 1125*	*2 1125*	*0*	*2 2700*	*2 2700*	*1 3375*
70	*3 60*	*2 450*	*2 450*	*3 420*	*1 6750*	*2 2700*	*2 2700*	*2 2700*	*2 1350*	*0*	*2 1350*	*1 6750*
73	*3 60*	*2 450*	*2 450*	*3 420*	*1 6750*	*2 2700*	*2 2700*	*2 2700*	*2 1350*	*2 1350*	*0*	*1 6750*
54	*4 24*	*3 180*	*3 180*	*2 900*	*2 1800*	*1 3375*	*1 3375*	*1 3375*	*1 3375*	*1 3375*	*1 3375*	*0*

Explanations:

a) The numbers 1 to 4 represent the number of steps or distance ***d.***

b) The numbers 24 to 30375 represent transition probabilities P_{ij} multiplied 50625 times.

c) We see that transition probability from symmetric structure GS_{34} to asymmetric structure GS_{54} $P_{34,54}$=1728**:**50625 ***is*** 1728**:**24=***72 times less*** than in opposite case, $P_{54,34}$=24**:**50625!

Asymmetric structures dominate. On the structural aspect are "asymmetrical" the ***0-symmetric*** and ***partially symmetric*** structures. In the system $\mathfrak{G}^{|V|=6}$ are eight 0-symmertic (5.73%), 140 partially symmetric structures (89.7%) and eight symmetric (edge- and polysymmetric) structures. Relative occurrence frequency of "asymmetric" structures, notably of *0*-symmetric structures, *increase* successfully by enlargement of degree |*V*|.

4.4. Structural successions as dynamic or evolutional phenomena

Exist real systems where their functioning can be expressed by gradual changes of structure in time. If the structures ***GS*** of the system $\mathfrak{G}^{|V|}$ are treated as ***states S of real system*** then a such succession, $SF= {}^{F1:}GS_0 \rightarrow {}^{F2:}GS_1 \rightarrow {}^{F3:}GS_2 \rightarrow \ldots \rightarrow {}^{Ft:}GS_{t-1} \rightarrow GS_t$ constitutes a ***dynamic or evolutional phenomenon,*** generated by morphisms.

Let the addition- or elimination operations **{*f*}** be treated as certain ***input impacts.*** Let morphisms as the sets of disjunctive operations $F_n=\{f_1 \vee \ldots \vee f_q\}_n$ form a ***class of effects*** **F.**

Let the values of structural and probabilistic characteristics form an ***output class*** **Y.** If the set $\{GS^{|V|}\}$ of all structures in the system $\mathfrak{G}^{|V|}$ be treated as a ***state class*** **S,** then the set **Y** of all the output values will represent a ***phase space*** of this system. An every time moment $t \in \mathbf{T}$ the system receives an input impact $f_t \in \mathbf{F}$, which changes the state S_{t-1} as a current step of succession, to a next state S_t. Each input impact f_t belongs also to a certain ***guide class*** $\mathbf{X}$, $f_t \in \mathbf{F} \subset \mathbf{X}$, which determines the possible change of a current state S_{t-1}. The selection of the class **F** can be ***random,*** or on the contrary related with a ***functional objective Z*** of the system.

The current values $y_t \in \mathbf{Y}$ of output set characterize a system state S_t, $y_t=\lambda(S_t)$. Knowing of the state S_t and fixing of an input impact $f_{t'} \in F_n$ is necessary and sufficient for determination of the state $S_{t'}=\varphi(S_t, f_{t'})$, always if $t<t'$.

It should also be noted, that our concept of ***dynamic system DYS*** is more concrete than the well-known concept of ***stationary (causal) system*** where each "present moment" ("past") changes to a "prospective moment" ("future"). In the case of dynamic system ***DYS***, it can be based on the system $\mathfrak{G}^{|V|}$, where:

a) one and the same "past" can be changed to various different "futures";
b) different (various) "pasts" can be changed to one and the same common "future".

Let us apply the concepts of the system of structural changes in $\mathfrak{G}^{|V|}$ to the classical concepts of the dynamic systems [14].

Propositions 4.10. Correspondence between attributes of the ***system*** $\mathfrak{G}^{|V|}$ and classical postulates of ***dynamic system DYS*****:**

P4.10.1. Correspondences of the sets:

a) The set $\{GS^{|V|}\}$ of all the structures in $\mathfrak{G}^{|V|}$ corresponds to a ***state class*** **S**;
b) the set $\{t\}$ of all the steps of successions corresponds to an ***ordered set of time moments (or stages)*** **T**;
c) the set **{*f*}** of *f*-operators corresponds to a ***effects class*** **F**;
d) the effects class **F** belong to ***guide class*** **X**;
e) the set of structural and probabilistic characteristics corresponds to a ***output class*** **Y** of the system ***DYS***.

P4.10.2. There exists an ***output mapping*** λ**:** $\mathbf{T} \times \mathbf{S} \rightarrow \mathbf{Y}$ that determines the output values $y_t \in \mathbf{Y}$ by each state S_t.

P4.10.3. The set of possible input impacts **X** be ***contracted*** to the classes of actual effects **F**.

P4.10.4. There exists an ***objective function*** μ**: T×Z×X→F,** such that at every time moment (stage) $t \in \mathbf{T}$ on the basis of *f*-impacts $f \in \mathbf{X}$ such a class $F_n \in \mathbf{F}$ is selected, which could help reach a ***functional objective*** **Z** in the best possible way. Functional aim **Z** is realized in the form of an aim state ZS_j or behaviour criterion ΔZ (in the form of monotonous chains).

P4.10.5. The main attribute of dynamic system ***DYS*** is a ***succession function*** φ**: T×S×F→S,** where its values are states $\mathbf{S}=\varphi(t';t,S,f)\in\mathbf{S}$, in which the system ***DYS*** is at a time moment (stage) $t' \in \mathbf{T}$, if at the preceding time moment $t \in \mathbf{T}$ it was in the preceding state $S \in \mathbf{S}$ and has an effect of the input impact $f \in F_n \subset \mathbf{X}$. The function φ has the following classical characteristics:

a) ***direction of time proceeding:*** function φ is determined for every t, where $t<t'<t''\ldots$;

b) ***a semi-group characteristic:*** by every $t<t'<t''$, every $S \in \mathbf{S}$ and every $f \in \mathbf{F}$ there is $\varphi(t'';t,S,f)=\varphi(t'';t',\varphi(t';t,S,f),f)$;

c) ***causality:*** if $f,f' \in F_n \subset \mathbf{F} \subset \mathbf{X}$, then $\varphi(t';t,S,f)=\varphi(t';t,S,f')$;

d) ***compatibility:*** equality $\varphi(t;t,S,f)=S$ is valid in the case of every $t \in \mathbf{T}$, every $S \in \mathbf{S}$ and every $f \in \mathbf{F}$.

Now we can formulate some corollaries.

Corollaries 4.4. On the ***successions SF*** and discrete ***dynamic system DYS:***

C4.4.1. Succession ***SF*** of structural changes represents a ***dynamic or evolutional process.*** Forming of succession constitutes a dynamic process itself.

C4.4.2. Discrete dynamic system ***DYS*** is ***stationary,*** if there exists a succession function φ. According to P5.4.4 the system ***DYS*** is ***teleological,*** if this does not hold, then ***DYS*** is ***stochastic.***

C4.4.3. The set of system states **S** of ***DYS*** constitute a ***factor space.***

C4.4.4. The various values of output characteristics **Y** of states form a ***phase space*** of ***DYS.***

C4.4.5. The triplet (t,S,y), $t \in \mathbf{T}$, $S \in \mathfrak{S}^{|V|}$, $y \in \mathbf{Y}$, represents an ***event*** and **T×S×Y** is an ***event space*** of ***DYS.***

In principle any questions can be asked about elementary changes. *How does differ semiotic model (text)* $\mathbf{SM}^{adj}$ *of adjacent structure* $\mathbf{GS}^{adj}$ *from semiotic model* **SM** *of initial structure* ***GS****?*

This chapter should be takes as an addition and application of presented structural treatment.

The successions as dynamical phenomena are applied to *simulation the evolution of lichens communities* [16] and other processes.