3. STRUCTURE MODEL AND ISOMORPHISM PROBLEM

Structure model is also a ***canonical presentation*** of a graph. The problem of canonical presentation was established probably by Lazlo Babai [1, 2] in 1977th. It means the presentation of graph in a certain form, preferably *with exactness of isomorphism.* ***Isomorphism problem*** consists in design an algorithm that recognizes the isomorphism of two objects. ***Isomorphism*** (Greek word ιισοσ – same; μορφε – form) constitutes *a one-to-one correspondence* between *structures* of objects [20, 25]. Such a one-to-one correspondence can only exist between abstract, idealized objects, which preserve the structure, i.e. relations, ordering, topology etc., of the systems.

3.1. Structure model as canonical presentation of a graph

As a rule, graphs canonized on the basis of *polynomials, spectra* [5], *3-cubecodes* [15] and other *global invariants* [9]. Unfortunately, such canonization does not contain the necessary information about the structure of a graph, this is not modeling. It is suggested to use also *local invariants*, such as *density, paths, cycles, cliques* and other [46]. We show that the binary signs are suitable local invariants and structure model can be considered as the "text" of structure.

Proposition 3.1. Structure model **SM** is a ***canonical presentation*** of the graph with exactness of *binary signs, structural attributes, positions and isomorphism.*

Example 3.1. Structure of Boris Weisfeiler's [45, p. 166 (a)] a *strongly regular* graph ***Wei*** recognizable and presentable on the basis of basic binary signs:

```
                          A:-2.8.20; B:-2.8.19; C:-2.8.18;
                          D:+2.7.13; E:+2.7.14; F:+2.7.15.

|1  1| 2  2|3  3|4  4| 5| 6  6|7  7|8  8| 9  9|10 10|11|12|13|14 14|15|        ui  k
20 24|12 14|1  2|9 19| 6|10 16|8 18|4  7|11 17|13 15|23| 3|22|21 25| 5|  i ABCDEF  *
|0  F| C  C|C  B|F  C| C| B  F|C  E|F  C| E  F| E  C| B| F| F| F  C| F| 20 039039  1
    0| C  C|B  C|C  F| C| F  B|E  C|C  F| F  E| C  E| B| F| F| C  F| F| 24 039039  1
     | 0  F|F  C|C  C| B| F  C|B  F|B  F| E  C| F  E| F| C| F| F  C| E| 12 039039  2
          0|C  F|C  C| B| C  F|F  B|F  B| C  E| E  F| F| C| F| C  F| E| 14 039039  2
           |0  F|F  C| E| F  C|F  B|F  E| F  C| F  C| C| F| B| C  C| E|  1 039039  3
              0|C  F| E| C  F|B  F|E  F| C  F| C  F| C| F| B| C  C| E|  2 039039  3
               |0  F| E| F  C|F  E|F  C| B  F| F  C| F| B| E| C  B| C|  9 039039  4
                  0| E| C  F|E  F|C  F| F  B| C  F| F| B| E| B  C| C| 19 039039  4
                   | 0| C  C|B  B|B  B| E  E| F  F| F| F| C| F  F| C|  6 066066  5
                      | 0  B|F  E|F  E| B  E| B  E| B| B| C| E  E| C| 10 066066  6
                           0|E  F|E  F| E  B| E  B| B| B| C| E  E| C| 16 066066  6
                            |0  C|F  B| F  B| C  C| E| C| B| C  E| E|  8 066066  7
                               0|B  F| B  F| C  C| E| C| B| E  C| E| 18 066066  7
                                |0  C| C  C| B  E| C| E| E| E  B| B|  4 066066  8
                                   0| C  C| E  B| C| E| E| B  E| B|  7 066066  8
                                    | 0  B| E  F| C| B| B| E  C| F| 11 066066  9
                                         0| F  E| C| B| B| C  E| F| 17 066066  9
                                          | 0  B| B| C| E| B  F| B| 13 066066 10
                                               0| B| C| E| F  B| B| 15 066066 10
                                                | 0| E| D| F  F| E| 23 066147 11
                                                   | 0| E| F  F| D|  3 066147 12
                                                      | 0| B  B| B| 22 093174 13
                                                         | 0  E| A| 21 147066 14
                                                              0| A| 25 147066 14
                                                               | 0|  5 255174 15
```

General invariants and measures (values):

Symmetry	*/V/*	*/R/*	*K*	*N*	*SVV*	*SV*	*SRV*	*HR*	*SR*
Partial symmetry	**25**	**300**	**15**	**154**	$1^{5}2^{10}$	*0.1723*	$1^{20}2^{128}4^{6}$	*2.1576*	***0.1290***

Specified invariants and measures of ***Wei*** and its complement ***WeiC***:

G	*/E/*	*k*	*N^{+}*	*N^{-}*	*P*	*CL*	*Girth*	*DM*	*SEV^{+}*	*SE*	*TRA*	*BRA*
WEI	150	1	80	74	6	4	3	2	$1^{12}2^{67}4^{1}$	*0.1310*	*1.000*	*0*
WEIC	150	1	74	80	6	4	3	2	$1^{8}2^{61}4^{5}$	*0.1494*	*1.000*	*0*

Explanations:

a) From structure model read out that ***Wei*** is ***partially symmetric, strongly regular, triangular, 2-distance- and 12-degree regular.***

b) On the ground of only ***six binary signs*** is the 25×25 structure model decomposed by help ***u***- and ***s***-vectors to ***15 vertex positions*** *(orbits)* and ***115 partial models*** $\mathbf{SM}_{ki,kj}$.

c) 150 "non edges" of ***Wei*** form ***74 binary(–)positions***, where *–A* form a position with two elements, *–B* form 33 positions, among these 4 with one element and 29 with two elements, *–C* form 40 positions, 4 with one, 31 with two and 5 with four elements.

d) 150 edges of ***Wei*** form ***80 binary(+)positions***, where ***+D*** form 2 positions with one element, ***+E*** 32 positions, among these 4 with one, 27 with two and one with four elements, and ***+F*** form 46 position, 6 with one and 40 with two elements.

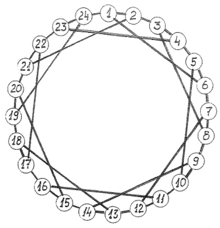
e) B. Weisfeiler [45] is one of these, who find that vertex orbits are essential attributes of graph structure. But the binary orbits he yet not perceives. He had constructed some strongly regular graphs which has grounded on the same pair signs, but are no isomorphic. On structural aspect: these differ from decompositions to binary positions.

f) Graph ***Wei*** and its complement ***WeiC*** is ***triangular, 2-distance-*** and ***12-degree regular.*** Complement ***WeiC*** is also ***strongly regular.***

NB! For analyzing the structure of ***Wei*** is suitable use its *sign graphs* $\boldsymbol{Wei}_{+2.7.13}$, $\boldsymbol{Wei}_{+2.7.14}$ and $\boldsymbol{Wei}_{+2.7.15}$.

Now is suitable to present a graph, where the recognition of its binary positions needed adjusted identification (Prop. 1.3).

Example 3.2. A simple ***polysymmetric*** graph ***Tev***, its *basic binary signs and structure model*:



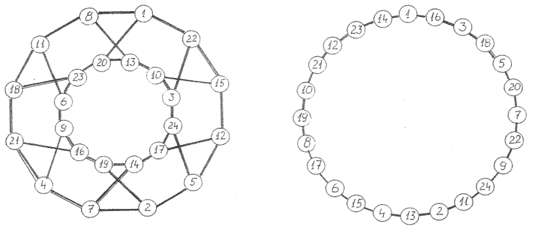
A:-5.18.23; B:-4.9.10; C:-4.8.8; D:-4.7.7; E:-3.8.9; F: -3.3.6; G:-3.4.3; H:-2.3.2;
I:+5.10.12; J:+5.12.15; K:+5.14.18.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	*i*	*ABCDEFGH*IJK	*k*
0	+K	H	E	H	+I	H	F	B	G	D	G	C	A	D	A	B	G	H	F	H	F	H	+J	1	22121336111	1
	0	+J	H	F	H	F	H	G	B	A	D	A	C	G	D	G	B	F	H	+I	H	E	H	2	22121336111	1
		0	+K	H	E	H	+I	H	F	B	G	D	G	C	A	D	A	B	G	H	F	H	F	3	22121336111	1
			0	+J	H	F	H	F	H	G	B	A	D	A	C	G	D	G	B	F	H	+I	H	4	22121336111	1
				0	+K	H	E	H	+I	H	F	B	G	D	G	C	A	D	A	B	G	H	F	5	22121336111	1
					0	+J	H	F	H	F	H	G	B	A	D	A	C	G	D	G	B	F	H	6	22121336111	1
						0	+K	H	E	H	+I	H	F	B	G	D	G	C	A	D	A	B	G	7	22121336111	1
							0	+J	H	F	H	F	H	G	B	A	D	A	C	G	D	G	B	8	22121336111	1
								0	+K	H	E	H	+I	H	F	B	G	D	G	C	A	D	A	9	22121336111	1
									0	+J	H	F	H	F	H	G	B	A	D	A	C	G	D	10	22121336111	1
										0	+K	H	E	H	+I	H	F	B	G	D	G	C	A	11	22121336111	1
											0	+J	H	F	H	F	H	G	B	A	D	A	C	12	22121336111	1
												0	+K	H	E	H	+I	H	F	B	G	D	G	13	22121336111	1
													0	+J	H	F	H	F	H	G	B	A	D	14	22121336111	1
														0	+K	H	E	H	+I	H	F	B	G	15	22121336111	1
															0	+J	H	F	H	F	H	G	B	16	22121336111	1
																0	+K	H	E	H	+I	H	F	17	22121336111	1
																	0	+J	H	F	H	F	H	18	22121336111	1
																		0	+K	H	E	H	+I	19	22121336111	1
																			0	+J	H	F	H	20	22121336111	1
																				0	+K	H	E	21	22121336111	1
																					0	+J	H	22	22121336111	1
																						0	+K	23	22121336111	1
																							0	24	22121336111	1

Explanation: Graph ***Tev*** has eleven *sign structures*, among these three sign(+)structures (i.e. substructures of ***Tev***) and eight sign(–)structures (i.e. substructures of complement ⌉***Tev***).

We show here two of these.

Example 3.3. ***Sign structures*** by sign *F: –3.3.6,* $\boldsymbol{Tev}_{p=-F}$ and by *A: –5.18.23,* $\boldsymbol{Tev}_{p=-A}$ (these not yet the position structures):



Explanation: Known, that the basic binary signs may not always be complete identifiers of vertex pairs, but on the ground of sign structures can be perfect the basic binary signs up to binary positions.

We show here the using of *productive binary signs* (PIA). The products E^n of small degree no give perfect information about the binary positions. In the case of graph ***Tev*** are the binary positions recognizable on the degrees ***n***=6 and ***n***=7 of matrix products E^n.

Example 3.4. *Union of the matrix-products* $\underline{E^{n=6}}$ and $E^{n=7}$ recognize the binary positions of ***Tev***:

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	258	84	243	75	239	65	191	42	150	33	130	32	107	33	108	42	139	65	173	75	201	84	248
2	258	0	248	84	201	75	173	65	139	42	108	33	107	32	130	33	150	42	191	65	239	75	243	84
3	84	248	0	258	84	243	75	239	65	191	42	150	33	130	32	107	33	108	42	139	65	173	75	201
4	243	84	258	0	248	84	201	75	173	65	139	42	108	33	107	32	130	33	150	42	191	65	239	75
5	75	201	84	248	0	258	84	243	75	239	65	191	42	150	33	130	32	107	33	108	42	139	65	173
6	239	75	243	84	258	0	248	84	201	75	173	65	139	42	108	33	107	32	130	33	150	42	191	65
7	65	173	75	201	84	248	0	258	84	243	75	239	65	191	42	150	33	130	32	107	33	108	42	139
8	191	65	239	75	243	84	258	0	248	84	201	75	173	65	139	42	108	33	107	32	130	33	150	42
9	42	139	65	173	75	201	84	248	0	258	84	243	75	239	65	191	42	150	33	130	32	107	33	108
10	150	42	191	65	239	75	243	84	258	0	248	84	201	75	173	65	139	42	108	33	107	32	130	33
11	33	108	42	139	65	173	75	201	84	248	0	258	84	243	75	239	65	191	42	150	33	130	32	107
12	130	33	150	42	191	65	239	75	243	84	258	0	248	84	201	75	173	65	139	42	108	33	107	32
13	32	107	33	108	42	139	65	173	75	201	84	248	0	258	84	243	75	239	65	191	42	150	33	130
14	107	32	130	33	150	42	191	65	239	75	243	84	258	0	248	84	201	75	173	65	139	42	108	33
15	33	130	32	107	33	108	42	139	65	173	75	201	84	248	0	258	84	243	75	239	65	191	42	150
16	108	33	107	32	130	33	150	42	191	65	239	75	243	84	258	0	248	84	201	75	173	65	139	42
17	42	150	33	130	32	107	33	108	42	139	65	173	75	201	84	248	0	258	84	243	75	239	65	191
18	139	42	108	33	107	32	130	33	150	42	191	65	239	75	243	84	258	0	248	84	201	75	173	65
19	65	191	42	150	33	130	32	107	33	108	42	139	65	173	75	201	84	248	0	258	84	243	75	239
20	173	65	139	42	108	33	107	32	130	33	150	42	191	65	239	75	243	84	258	0	248	84	201	75
21	75	239	65	191	42	150	33	130	32	107	33	108	42	139	65	173	75	201	84	248	0	258	84	243
22	201	75	173	65	139	42	108	33	107	32	130	33	150	42	191	65	239	75	243	84	258	0	248	84
23	84	243	75	239	65	191	42	150	33	130	32	107	33	108	42	139	65	173	75	201	84	248	0	258
24	248	84	201	75	173	65	139	42	108	33	107	32	130	33	150	42	191	65	239	75	243	84	258	0

Explanation: All the identifiers of vertex pairs (i.e. binary positions) here are complete. The matrix products $\underline{E^{n=6}}$ and $E^{n=7}$ are here united, because in both case exist the zero values.

It suitable to associate the basic binary signs with the results of matrix product E^n of this graph:

Example 3.5. Associating the basic and productive binary signs:

1	**1**		**2**	**3**	**4**	**5**	**6**			**7**			**8**			**9**	**10**	**11**
2	***-A***		***-B***	***-C***	***-D***	***-E***	***-F***			***-G***			***-H***			***+I***	***+J***	***+K***
3	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
4	108	107	42	32	33	243	191	201	173	150	139	130	65	75	84	**239**	**248**	**258**
5	*-A1*	*-A2*	*-B*	*-C*	*-D*	*-E*	*-F1*	*-F2*	*-F3*	*-G1*	*-G2*	*-G3*	*-H1*	*-H2*	*-H3*	***+I***	***+J***	***+K***
6	*1*	*1*	*2*	*1*	*2*	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*2*	*2*	*2*	***1***	***1***	***1***

Explanations: 1 – the ordering number of basic binary signs; 2 – basic binary signs (see example 2); 3 – the ordering number of productive binary signs; 4 – productive binary signs (see example 4); 5 – marking of productive binary signs (see example 6); 6 – the last row is there the *frequency vector* for all the rows (vertices) of structure model. The number of basic binary signs is 11, the number of complete binary signs is 18. Perfected binary sign constitutes a quintuplet $\pm \boldsymbol{d.n.q.e}^{n}_{ij}$, where the last represents the perfecting (see the fives row).

Example 3,6. The ***complete structure model*** **SM*** of ***Tev***:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	*i*	*k*
0	**K**	*H3*	*E*	*H2*	**I**	*H1*	*F1*	*B*	*G1*	*D*	*G3*	*C*	*A2*	*D*	*A1*	*B*	*G2*	*H1*	*F3*	*H2*	*F2*	*H3*	**J**	1	*1*
	0	**J**	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	*A2*	*C*	*G3*	*D*	*G1*	*B*	*F1*	*H1*	**I**	*H2*	*E*	*H3*	2	*1*
		0	**K**	*H3*	*E*	*H2*	**I**	*H1*	*F1*	*B*	*G1*	*D*	*G3*	*C*	*A2*	*D*	*A1*	*B*	*G2*	*H1*	*F3*	*H2*	*F2*	3	*1*
			0	**J**	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	*A2*	*C*	*G3*	*D*	*G1*	*B*	*F1*	*H1*	**I**	*H2*	4	*1*
				0	**K**	*H3*	*E*	*H2*	**I**	*H1*	*F1*	*B*	*G1*	*D*	*G3*	*C*	*A2*	*D*	*A1*	*B*	*G2*	*H1*	*F3*	5	*1*
					0	**J**	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	*A2*	*C*	*G3*	*D*	*G1*	*B*	*F1*	*H1*	6	*1*
						0	**K**	*H3*	*E*	*H2*	**I**	*H1*	*F1*	*B*	*G1*	*D*	*G3*	*C*	*A2*	*D*	*A1*	*B*	*G2*	7	*1*
							0	**J**	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	*A2*	*C*	*G3*	*D*	*G1*	*B*	8	*1*
								0	**K**	*H3*	*E*	*H2*	**I**	*H1*	*F1*	*B*	*G1*	*D*	*G3*	*C*	*A2*	*D*	*A1*	9	*1*
									0	**J**	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	*A2*	*C*	*G3*	*D*	10	*1*
										0	**K**	*H3*	*E*	*H2*	**I**	*H1*	*F1*	*B*	*G1*	*D*	*G3*	*C*	*A2*	11	*1*
											0	**J**	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	*A2*	*C*	12	*1*
												0	**K**	*H3*	*E*	*H2*	**I**	*H1*	*F1*	*B*	*G1*	*D*	*G3*	13	*1*
													0	**J**	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	*A1*	*D*	14	*1*
														0	**K**	*H3*	*E*	*H2*	**I**	*H1*	*F1*	*B*	*G1*	15	*1*
															0	**J**	*H3*	*F2*	*H2*	*F3*	*H1*	*G2*	*B*	16	*1*
																0	**K**	*H3*	*E*	*H2*	**I**	*H1*	*F1*	17	*1*
																	0	**J**	*H3*	*F2*	*H2*	*F3*	*H1*	18	*1*
																		0	**K**	*H3*	*E*	*H2*	**I**	19	*1*
																			0	**J**	*H3*	*F2*	*H2*	20	*1*
																				0	**K**	*H3*	*E*	21	*1*
																					0	**J**	*H3*	22	*1*
																						0	**K**	23	*1*
																							0	24	*1*

Explanations:

- **a)** Graph ***Tev*** is ***3-degree-*** and ***6-girth-regular***, its *complement* ***TevC*** is *20-degree-, 2-distance-, 3-girth-regular* and also *polysummetric*.
- **b)** From 6-girth regularity concludes that ***Tev*** is ***bipartite***, in present case parts with even- and odd-numbered vertices.
- **c)** As ***Tev*** is bipartite, but not *bi*-clique, then its complement ***TevC*** consists of two mutually connected *12-cliques* and is thus ***12-clique-regular***. These cliques correspond to parts of ***Tev***.
- **d)** The number *N* of *position- and adjacent structures* is 18, their powers coincide in cases ***Tev*** and its complement ***TevC***.
- **e)** 23x24**:**2 = 276 vertex pairs of ***Tev*** form 18 ***binary positions***, where by 240 "non-edges" be formed 15 binary(–)positions with 12 and 24 elements. 36 adjacent vertex pairs form three binary(+)positions, ***+I***, ***+J*** and ***+K***, with 12 elements and are recognized by the basic binary signs.
- **f)** ***Position-structures*** of ***Tev*** are mostly ***bisymmetric*** *(two binary positions)*, ***2-clique-regular*** and mutually ***isomorphic***. Position structures by *–B, –D, –H1, –H2* and *–H3* constitute ***girths***.
- **g)** 276 possible ***adjacent graphs*** converged to 15 ***adjacent super-structures*** and to three ***adjacent sub-structures***.

The *basic binary signs* not lose its meaning, these characterize the relationships between vertices, the belonging of vertex pairs to (assemblage of) paths or girths with corresponding size etc. These are needed for characterizing of the structure as a whole.

Conclusion 3.1. *Time complexity of structure's recognition* depends only at the number of vertex pairs.

3.2. Structural equivalence and graph isomorphism

Isomorphism is *an invertible morphism*, which has *an opposite morphism*, such that their product is *the unity morphism*. A topological isomorphism is called a *homeomorphism*.

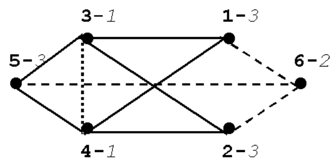
The *graph isomorphism problem* first came into prominence in 1857, when Arthur Cayley [4] reported his research on organic isomers. On structural aspect is it the problem of comparing the structural models. We demonstrate that the structure model and isomorphism problem are closely related.

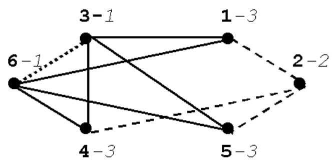
Isomorphic graphs have the same structure, which is expressed in the form of structural equivalence of models.

Proposition 3.2. On the relationships between isomorphism and structural equivalence of graphs:

1) Isomorphism is a one-to-one correspondence between elements where an isomorphic mapping from graph G_A to graph G_B is a bijection $\varphi: V_A \rightarrow V_B$ [1].
2) Isomorphism recognition does not recognize the structure, but the structure model recognizes the structure with exactness up to isomorphism [31].
3) Structural equivalence is a coincidence or bijection on the level of binary signs, binary- and element positions [34].
4) Recognition of the positions by the structure model is more effective than detecting the orbits on the ground of the group ***AutG***.

Example 3.7. Graphs G_A and G_B, their basic binary signs and structure models $\mathbf{SM}_A$ and $\mathbf{SM}_B$:





A:-2.5.7; B:-2.5.6;
C:+2.3.3; D:+2.5.7; E:+3.6.10.

A:-2.5.7; B:-2.5.6;
C:+2.3.3; D:+2.5.7; E:+3.6.10.

1	1	2	3	3	3		u_i	k	s_i
3	4	6	1	2	5	i	ABCDE		123
0	D	-B	C	C	C	3	01310	1	103
	0	-B	C	C	C	4	01310	1	103
		0	E	E	E	6	02003	2	003
			0	-A	-A	1	20201	3	210
				0	-A	2	20201	3	210
					0	5	20201	3	210

≈

1	1	2	3	3	3		u_i	k	s_i
3	6	2	1	4	5	i	ABCDE		123
0	D	-B	C	C	C	3	01310	1	103
	0	-B	C	C	C	6	01310	1	103
		0	E	E	E	2	02003	2	003
			0	-A	-A	1	20201	3	210
				0	-A	4	20201	3	210
					0	5	20201	3	210

Explanations:

a) Different graphs G_A and G_B have equivalent structure models $\mathbf{SM_A} \approx \mathbf{SM_B}$! This means that the structures are ***equivalent*** and the graphs ***isomorphic*** $G_A \cong G_B$.

b) The element pairs are divided to ***five binary positions*** ΩR_n, wherein the adjacent elements or "edges" to *three binary(+)positions (full line, a dotted, dashed-line)* that coincides with binary signs ***C, D, E*** correspondingly. The structural elements are divided to ***three positions*** ΩV_k.

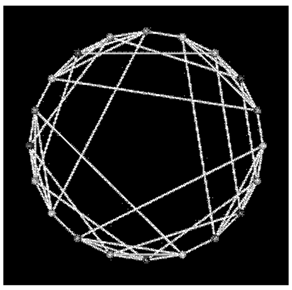
c) The column u_i of model consists of the ***frequency vectors***, which for the element i show its relations with other elements. On the basis of vectors u_i are arranged the positions in model.

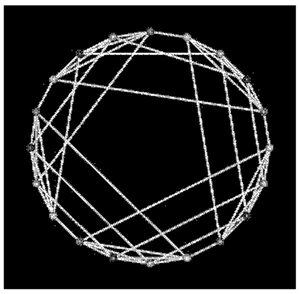
d) The column s_i of model consists of the ***position vectors*** that represent the connections of element i with elements in corresponding positions k. If on the framework of frequency vectors arises differences of position vectors, then by lasts does a complementary partition into classes.

Proposition 3.3. For recognition the equivalence of structure models *A* and *B* is necessary and sufficient to establish: 1) coincidence of the *sequences of binary signs* $\{\pm d.n.q._{ij}\}_A$ and $\{\pm d.n.q._{ij}\}_B$; 2) coincidence of the *frequency vectors* $\{u_i\}_A$ and $\{u_i\}_B$; 3) coincidence of the *position vectors* $\{s_i\}_A$ and $\{s_i\}_B$.

In following we look the models of the especially for isomorphism testing constructed two *graphs*.

Example 3.8. Poly-symmetric graphs Pra_A and Pra_B, their basic and adjusted binary signs and models:





Common basic pair signs of $\boldsymbol{Pra_A}$ and $\boldsymbol{Pra_B}$:

A:-3.8.10; B:-3.6.7; C:-2.4.4; D:-2.3.2; **E:+2.4.6**; **F:+3.8.16.**

Adjusted by matrix product $\boldsymbol{E}^{n=5}$ binary signs and complete structure model of $\boldsymbol{Pra_A}$:

Marking the basic binary signs	0	-A	-B	-C		-D	E		F
Productive binary signs e^5	180	125	110	165	160	80	231	233	210
Adjusted binary signs	0	-A	-B	-C1	-C2	-D	**E1**	**E2**	**F**

```
|   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20|   i  ABCCDEEF   k
|   0  E2  E1  E1   F  C2  C1  C1   F  C2  C1  C1   D   A   B   B   D   A   B   B|   1  24422212   1
        0  E1  E1  C2   F  C1  C1  C2   F  C1  C1   A   D   B   B   A   D   B   B|   2  24422212   1
            0  E2  C1  C1   F  C2  C1  C1   F  C2   B   B   D   A   B   B   D   A|   3  24422212   1
                0  C1  C1  C2   F  C1  C1  C2   F   B   B   A   D   B   B   A   D|   4  24422212   1
                    0  E2  E1  E1   D   A   B   B   F  C2  C1  C1   A   D   B   B|   5  24422212   1
                        0  E1  E1   A   D   B   B  C2   F  C1  C1   D   A   B   B|   6  24222212   1
                            0  E2   B   B   D   A  C1  C1   F  C2   B   B   A   D|   7  24222212   1
                                0   B   B   A   D  C1  C1  C2   F   B   B   D   A|   8  24222212   1
                                    0  E2  E1  E1   A   D   B   B   F  C2  C1  C1|   9  24222212   1
                                        0  E1  E1   D   A   B   B  C2   F  C1  C1|  10  24222212   1
                                            0  E2   B   B   A   D  C1  C1   F  C2|  11  24222212   1
                                                0   B   B   D   A  C1  C1  C2   F|  12  24222212   1
                                                    0  E2  E1  E1  C2   F  C1  C1|  13  24222212   1
                                                        0  E1  E1   F  C2  C1  C1|  14  24222212   1
                                                            0  E2  C1  C1  C2   F|  15  24222212   1
                                                                0  C1  C1   F  C2|  16  24222212   1
                                                                    0  E2  E1  E1|  17  24222212   1
                                                                        0  E1  E1|  18  24222212   1
                                                                            0  E2|  19  24222212   1
                                                                                0|  20  24222212
```

Adjusted by matrix product $\boldsymbol{E}^{n=7}$ binary signs and complete structure model of $\boldsymbol{Pra_B}$:

Basic binary signs	0	-A	-B		-C			-D	E		F
Productive signs e^7	4410	3437	3276	3277	4081	4088	4011	3010	4831	4803	4445
Adjust. binary signs	0	-A	-B1	-B2	-C1	-C2	-C3	-D	**E1**	**E2**	**F**

```
|   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20|   i  ABBCCCDEEF   k
|   0  E1  E2  E1   F  C1  C2  C3   F  C3  C2  C1   D  B2  B1   A   D   A  B1  B2|   1  2222222212   1
        0  E1  E2  C3   F  C1  C2  C1   F  C3  C2   A   D  B2  B1  B2   D   A  B1|   2  2222222212   1
            0  E1  C2  C3   F  C1  C2  C1   F  C3  B1   A   D  B2  B1  B2   D   A|   3  2222222212   1
                0  C1  C2  C3   F  C3  C2  C1   F  B2  B1   A   D   A  B1  B2   D|   4  2222222212   1
                    0  E1  E2  E1   D   A  B1  B2   F  C1  C2  C3   A   D  B2  B1|   5  2222222212   1
                        0  E1  E2  B2   D   A  B1  C3   F  C1  C2  B1   A   D  B2|   6  2222222212   1
                            0  E1  B1  B2   D   A  C2  C3   F  C1  B2  B1   A   D|   7  2222222212   1
                                0   A  B1  B2   D  C1  C2  C3   F   D  B2  B1   A|   8  2222222212   1
                                    0  E1  E2  E1   A  B1  B2   D   F  C3  C2  C1|   9  2222222212   1
                                        0  E1  E2   D   A  B1  B2  C1   F  C3  C2|  10  2222222212   1
                                            0  E1  B2   D   A  B1  C2  C1   F  C3|  11  2222222212   1
                                                0  B1  B2   D   A  C3  C2  C1   F|  12  2222222212   1
                                                    0  E1  E2  E1  C3   F  C1  C2|  13  2222222212   1
                                                        0  E1  E2  C2  C3   F  C1|  14  2222222212   1
                                                            0  E1  C1  C2  C3   F|  15  2222222212   1
                                                                0   F  C1  C2  C3|  16  2222222212   1
                                                                    0  E1  E2  E1|  17  2222222212   1
                                                                        0  E1  E2|  18  2222222212   1
                                                                            0  E1|  19  2222222212   1
                                                                                0|  20  2222222212   1
```

Explanations:

a) $\boldsymbol{Pra_A}$ and $\boldsymbol{Pra_B}$ both are ***5-degree-, 4-girth-, 4-clique*** regular and have *six common basic binary signs. 4-clique regularity* expressed by existence the five *4*-cliques, what are in structure model showy as signs ***E***.

b) ***Pra_A differ*** at ***Pra_B*** by the number of adjusted binary positions, *eight* and *ten* correspondingly. Consequently, structures ***Pra_A*** and ***Pra_B*** are ***non equivalent*** and its graphs ***non isomorphic.***
c) Both graphs have three binary(+)positions ***E1, E2*** and ***F*** with power 20.
d) Graph ***Pra_A*** has five pair(−)positions: by *–A, −C2,* and *−D* with power 20, and by *−B* and *−C1* with power 40.
e) The ***complement $PraC_A$*** of ***Pra_A*** has pair signs *–A:-2.14.68, -B:-2.12.47,* ***C:+2.10.35, D:+2.10.36, E:+2.11.44, F:+2.12.48*** and is ***triangular*** and ***14-degree regular***.
f) Graph ***Pra_B*** has seven pair(−)positions with power 20.

On the graphs ***Pra_A*** and ***Pra_B*** give interest also their ***position structures*** that are more than *2-degee-regular.* Such are position structures of ***Pra_A*** by positions *−B* and *–C1.* For example, position structure ***$Pra_{A(-B)}$*** is (+)***symmetric, 5-partite*** and ***4-girth regular***, where the parts of ***$Pra_{A(-B)}$*** correspond to *4*-cliques of ***Pra_A***: **I** – vertices **1,2,3,4; II** – vertices **5,6,7,8; III – 9,10,11,12; IV – 13,14,15,16**; **V – 17,18,19,20.**

Conclusion 3.2. *Time complexity of ascertaining the structural equivalence* depends only at the number of vertex pairs and is polynomial.

3.3. Isomorphism recognition of strongly symmetric graphs

It is possible to construct such *bisymmetric and strongly regular graphs* that have very small binary graphs in case of large number of vertices. We call these ***strongly symmetric graphs.***

Constructed by M. Nechepurenko, M. Klin [19] et al in Siberian *strongly symmetric graphs* ***Sib_A*** and ***Sib_B*** with 40 vertices have ***common binary signs***: *–A:–2.6.8* (complement has ***+B:+2.20.142***) and ***+B:+2.4.6*** (the complement has *–A:–2.20.144*). From binary signs conclude that ***Sib_A*** and ***Sib_B*** are *4-clique-, 2-distance-* and *12-degree regular.* From coincidence the binary signs of ***Sib_A*** and ***Sib_B*** conclude the *coincidence of the symmetry properties.*

As in case of strongly regular graphs the product identification ***PIA*** no works, we must use another methods of deep-identification. By *high identification* (P1.3.1) the *second degree binary signs* of ***Sib_A*** and ***Sib_B*** are $–A^{m=2}$*=–3.18.48* and ***$+B^{m=2}$=+3.20.64***, and anew coincide. A binary graph of third degree ***$g_{ij}^{m=3}$*** no arise, it is empty ∅.

Now must be form by help the *local identification method* (P1.3.2) *local structure models* **$SM_{ij}^{m=2}$** for *second degree binary graphs* ***$g_{ij}^{m=2}$*** of ***Sib_A*** and ***Sib_B***. For this we open in both graphs a binary graph ***$g_{ij}^{m=2}$***, such that correspond to binary sign ***$+B^{m=2}$***.

Example 3.9. Binary signs the local structure models of second degree binary graphs ***$g_{ij}^{m=2}$*** of ***Sib_A*** and ***Sib_B*** correspondingly:

Binary signs of second degree binary graph ***$g^{m=2}$***$\subset$ ***Sib_A*** in local structure model **$SM_{ij}^{m=2}{}_A$**:
–A=–2.6.8; –B=–2.4.4; –C=–2.3.2; ***D=+2.4.6; E=+3.12.28; F=+3.20.46.***
Binary signs of second degree binary graph ***$g^{m=2}$***$\subset$ ***Sib_B*** in local structure model **$SM_{ij}^{m=2}{}_B$**:
–A=–2.6.8; –B=–2.4.4; ***C=+2.4.6; D=+3.12.24; E=+3.20.46.***

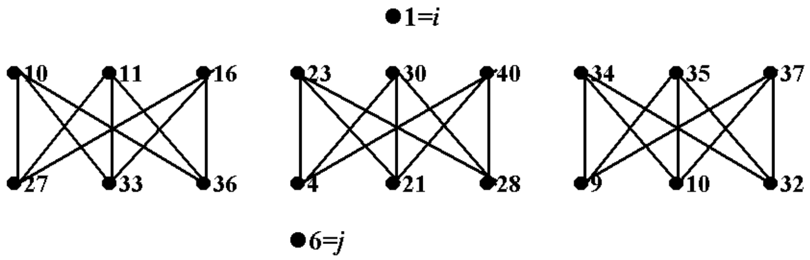
Explanations: **a)** From differences of binary signs conclude *non-isomorphism* of second degree binary graphs. **b)** From non-isomorphism the binary graphs conclude non-isomorphism of graphs ***Sib_A*** and ***Sib_B.***

Proposition 3.3. From non-isomorphism the binary graphs ***g_{ij}^A*** and ***g_{ij}^B*** of corresponding symmetric graphs ***G_A*** and ***G_B*** conclude ***non-isomorphism*** of ***G_A*** and ***G_B.***

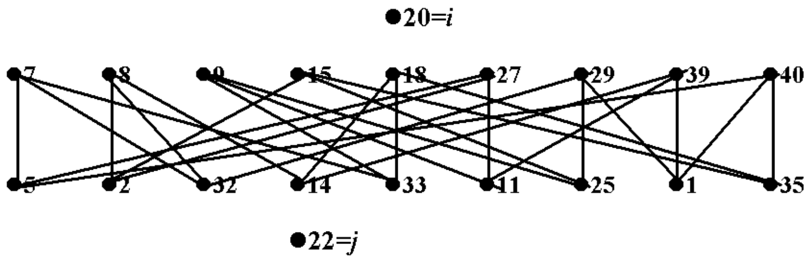
For illustrating the differences of ***Sib_A*** and ***Sib_B*** is suitable demonstrate second degree binary graphs.

Example 3.10. The kernels of second degree binary graphs $g^{m=2}{}_A$ and $g^{m=2}{}_B$ of very similar structures $\boldsymbol{Sib_A}$ and $\boldsymbol{Sib_B}$:

Kernel of $g_{1\text{-}6}{}^{m=2} \subset Sib_A$:



Kernel of $g_{20\text{-}22}{}^{m=2} \subset Sib_B$:



3.4. Outputs of two isomorphism algorithms

Only few isomorphism recognition algorithms give a canonical output of processing results. Usually be limited laconically with phrase "isomorphic" or "not isomorphic". We show the ***canonical output*** of two algorithms the isomorphism recognition. Recognition the orbits no belong to isomorphism problem. In historical journey so far, the question remains: *is the graph isomorphism problem in* ***P****?*

First, Dharvadker-Tevet polynomial algorithm [6] based on ***incomplete semiotic models*** $\boldsymbol{S_A}$ and $\boldsymbol{S_B}$, where the vertex classes $\boldsymbol{V_{Ak}}$ and $\boldsymbol{V_{Bk}}$ given on the level only of frequency vectors. Notable here are the following moments: 1) Transposition the rows $\boldsymbol{i}$ and columns $\boldsymbol{j}$ take place within vertex classes of corresponding partial matrices $\boldsymbol{S_{Ak}}$ and $\boldsymbol{S_{Bk}}$ and leaded to isomorphism recognition with exactness up to substitutions. 2) Isomorphism recognition and its time complexity are proved in detail.

Example 3.11. Canonical outputs of Dharwadker-Tevet isomorphism algorithm:

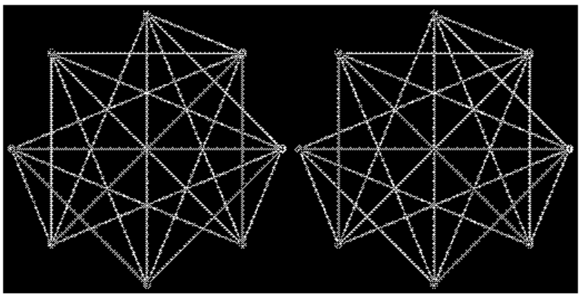
Incomplete structure model of $\boldsymbol{G_B}$:

Matrix *B*	1	8	7	3	6	4	5	2
1	-0.1.0	-2.8.21	+2.5.7	+2.5.7	+2.5.7	+2.5.7	+2.5.7	+2.5.7
8	-2.8.21	-0.1.0	+2.5.7	+2.5.7	+2.5.7	+2.5.7	+2.5.7	+2.5.7
7	+2.5.7	+2.5.7	-0.1.0	-2.7.16	-2.7.16	+2.4.5	+2.4.5	+2.4.5
3	+2.5.7	+2.5.7	-2.7.16	-0.1.0	-2.7.16	+2.4.5	+2.4.5	+2.4.5
6	+2.5.7	+2.5.7	-2.7.16	-2.7.16	-0.1.0	+2.4.5	+2.4.5	+2.4.5
4	+2.5.7	+2.5.7	+2.4.5	+2.4.5	+2.4.5	-0.1.0	-2.7.16	-2.7.16
5	+2.5.7	+2.5.7	+2.4.5	+2.4.5	+2.4.5	-2.7.16	-0.1.0	-2.7.16
2	+2.5.7	+2.5.7	+2.4.5	+2.4.5	+2.4.5	-2.7.16	-2.7.16	-0.1.0

Substitutions of graphs G_A and G_B:

Graph G_A	Graph G_B
4	1
5	8
1	7
2	3
3	6
7	4
8	5
6	2

Isomorphic graphs G_A and G_B;



Explanations: a) Isomorphism recognition of this algorithm and its ***polynomial*** time complexity are proved in detail. b) In this algorithm are the semiotic models S_A and S_B complete, because to their decomposition be suffice the exactness of frequency vectors. c) By this algorithm are recognized also canonically hardly recognizable graphs, for example, the non-isomorphism of *strongly symmetric* graphs Sib_A and Sib_B (see Example 3.10).

*

To canonical output of a graph in isomorphism algorithm of Blazej Podsiadlo [22] is its *biggest value* that no contain data about the graph, but enable to differentiate these, better as for example 3-cube-codes. It do no realized up to substitutions. To the canonical output belong *the biggest value* **<the biggest value>**, *the number of paths* **<paths>**, *the number of automorphisms* **<automorphisms>**, *the real time* **<treal>**.

<u>Examples 3.12.</u> A result of isomorphism algorithm, by Blazej Podsiadlo**:**

<example> <paths> <automorphisms> <treal> <the biggest value>

<1A> <720> <79> <0m0.074s>
1250779006614106429870497093178760201282717954568043451146532584283558626000476535 653
1208325182199136378607036959052271 00

<1B> <720> <34> <0m0.074s>
1250779006614106429870497093178760201282717954568043451146532584283558626000476535621
219280757394118382351094747106378100
Result: **NOT Isomorphic**

Explanations: a) It ensues on the rather great coincidence in the beginning of sums **83/121** or **68,60%** similar**.** b) It is a performance with two vertex symmetric graphs $\boldsymbol{Pra_A}$ and $\boldsymbol{Pra_B}$ (Example 3.8) that have common first degree pair signs and are very similar.

The “length” of value depends on the vertex number and coincidence on relation the “lengths” of intersection and full value. In original program comparison the sums do not exist. As I have experience with these graphs, the results seem logical and acceptable. Naturally, their essence needs to research.