SEMIOTIC MODELLING OF THE GRAPHS

John-Tagore Tevet

__Abstract:__ Semiotic modelling of graphs has emerged from the need to bring into order and to explain the general meaning of structure of graphs. Here is presented a way for recognition of the structure of graphs with exactness up to orbits (positions), isomorphism and other structural properties. It is implemented in the form of a semiotic model that enables to explain the essential properties of structure and their transformations.

1. INITIAL PRINCIPLES

1.1. Essence of structure

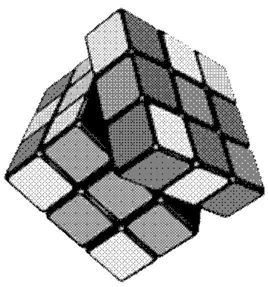
What's the common between such associations or connected sets as a ***system, structure***, as well a ***graph***. All three consist of *elements* and their *relationships*. The system has *many aspects* and depends from the *empirical properties* of the elements and relationships. In the system has essential role its ***function*** and ***structure***.

Structure is a specific characteristic of the associations, an inseparable attribute of all the really existing systemic objects. Structure must be the *constructional* or *organizational side* of the systemic object [20]. Unfortunately, is the concept of structure devalued to fuzzy adjective of any objects. Structure exists there where the relations between element pairs are recognizable [39, 40]. These relations are simple recognizable in case of chemical compounds, genetic formations, some networks etc.

Structure constitutes *an abstraction of the system*, its "skeleton", where its elements and relationships are lose its empirical properties but retain their qualitative distinctness in the form of different positions in the structure. Structure (Latin word *structura (inner)building*) is defined as a *connection, permanent relationship* or *organizational way* of system's elements [25]. Be argued that to the *model (interpreter, explicate)* of the structure is a ***graph.*** All the structural properties are explained on a graph, including the *positions*.

Concepts of *system, structure, position and graph* are easily and pictorially explainable on the ***Rubik's Cube.***

__Example 1.1.__ To this end, let's look at the Rubik's Cube and answer to two questions: 1) What kind of positions have the elements of cube? 2) With turning a layer of the cube changes its structure or system?



Answer 1. In Rubik's cube has each facet 9 elements, so on all the facets are 6x9 = 54 elements. Each facet has one element in the *middle*, four elements in the *edges* and four elements in the *angles*. Thus, the 6 elements of the cube represent a ***"middle position"***, 24 elements an ***"edge position"*** and 24 elements an ***"angle position"***.

Answer 2. With turning the layers of the cube ***changes its system**,* because the relationships between its empirical properties of the elements (i.e. colors) changed. However, the ***structure does not change***, because the ***positions are remained***, i.e. stay to the ***invariant***.

The *structure* of Rubik's cube can depict in the form of a ***graph*** (here can be remark that Rubik's cube as a system has also some aspects – its elements can be also its angles and edges). For present case, each element of this cube has four neighbors: "upper", "lower", "right", "left" and can be presented as a graph, where its 54 vertices divide to ***the three positions*** – to *"middle"*, *"edge"* and *"angle"* position. As a rule is every structure presentable in the form of a graph and is intimately related with ***invariance*** and ***isomorphism***.

How different the structure from a graph?

Propositions 1.1. On the *difference of the structure and a graph*:

P1.1. ***Isomorphic graphs have the same structure***, non-isomorphic different structures, in the other words, *structure is a complete invariant of isomorphic graphs.*

P1.2. If for the presentation of a graph is sufficient to present its adjacency matrix then *for recognition of structure* ***must the relationships between elements be identified with exactness up to the orbits (positions).***

The orbits are in the group theory known as *transitivity domain of automorphisms* or *eqiuvalence classes*, we call it to ***positions.*** The difference consists only in detection techniques. Identifiers of the element pairs called ***binary signs***. An ordered system of binary signs forms the ***semiotic model of structure***, which presents the structure with exactness up to positions and isomorphism [28 - 40].

This problem is ***heuristic***. To research objects of Heuristics are the *thinking and creative processes, complicated systems and their formalization efforts*. Modern heuristic is connected with the problems of ***artificial intelligences***. In case of complicated problems of the graphs can be their treatment with exact mathematical methods impossible. Heuristics arise as soon as there appear an alternative – and the solving almost always subjective. As we see heuristics is unavoidable for IT and other fields.

To one of heuristic methods is ***semiotics.*** It is a discipline of the *signs and sign systems* that study the phenomena of the *meaning, communication- and interpretation.* The development and implementation of majority the semiotic methods based on the investigation of such systems, which have sufficiently clearly expressed *structure* and sufficiently clear means for *expression of their attributes.* Thereafter, I will try to show that these conditions satisfy the graphs. Semiotic investigations enable formalizing the new objects and in the border areas emerged disciplines, as well as for research of known objects on a *new aspect*.

Sign of a sensually perceptible object (a thing, phenomenon, condition, events) that *represents, sign or describe* from its self different object, its *properties, meaning or sense.* You might say that sign is the *concentrate of an object,* by help which stored, processed and transmitted the information. An object is a sign only in a sure relation with the other signs which partake in the same process and are the same type. In case of a sign is concretized its *meaning* (i.e. indication function, which it represents), and its *sense* (i.e. with a sign associated content of sense). Sign is related with ***cognition*** and ***thinking***. Sign is an essential characteristic of an object. For these reasons is here suitable use the ***semiotic modeling***.

Semiotics is characterized with pluralism. The semiotics exist many, in the areas of arts and science. By W. Nöth [21] exist a ***semiosphere***, whereto belong from the semiotics of culture to the computer semiotics. One of the first semiotics was *Semiotics of Mathematics* [12]. Semiotics can be related with *computing and artificial intelligence.* Semiotics is *interdisciplinary.* Semiotic modeling can be one of the many kinds of object-oriented semiotics.

To structural signs are the ***semiotic invariants*** of the graphs [36]. Thus, for structure recognition must the element pairs in the *structure model* identified with exactness up to ***binary positions***, i.e. *the positions of element pairs in the structure).*

1.2. Semiotic model of structure

Semiotic model of structure **SM** is an ordered system of identifiers of vertex pairs, i.e. system of binary signs.

Let a vertex pair ***ij*** can be identified by a specific "relationship" between their in the form of *intersection of neighborhood*, $N_i \cap N_j$, as a partial graph, called ***binary graph***. Corresponding ***semiotic identification algorithm SIA*** find all the binary graphs and determines their *semiotic invariants*, in the form of ***basic binary sign*** [23].

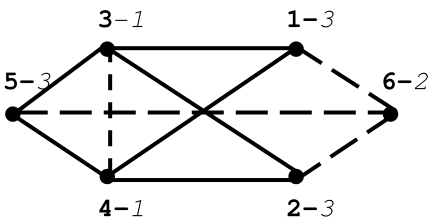
Semiotic Identification Algoritm *(SIA).* OPERAND: *List the adjacent vertices* ***L.*** ALGORITHM: 1) Fix an element ***i*** and form its neighborhood N_i, where the elements, connected with ***i***, are divided according to distance ***d*** to entries C_d. 2) Fix an element ***j*** and form its neighborhood N_j by condition (1). 3) Fix the intersection $N_i \cap N_j$, as a *binary graph* g_{ij}, and fix its invariants in the form of a binary sign $\pm d.n.q._{ij}$. 4) Realize (1) to (3) for each pair ***ij***, $i,j \in [1, |V|]$. 5) Obtained preliminary semiotic model. Fix for each vertex (row) ***i*** its *frequency vector* u_i of pair signs. 6) Decompose the preliminary model **SM** by *frequency vectors* u_i lexicographically to partial models SM_k. 7) In the framework of SM_k decompose the rows and columns by *class vectors* s_i lexicographically to complementary partial models SM_k. 8) Repeat (7) up to complementary decomposing no arise. RESULTS: **a)** ***Semiotic model*** **SM**; **b)** *The lists of vertices* $\{B_{ij}\}$ *of binary graphs.*

Explanation: The ***basic binary sign*** is a quadruplet $\pm d.n.q._{ij}$, where ***+d*** show collateral- and ***–d*** ordinary distance between vertices v_i and v_j, ***n*** – number of vertices and ***q*** – number of edges, in this binary graph g_{ij}.

It has been suggested the binary sign to *measure (size)* called. Indeed, it has the properties of measure. But yet, this is pointless, because in present case, needed a ***description*** of the condition of vertex pairs in the structure.

Example 1.2. Graph ***G***, its binary signs and ***model*** **SM**:

A:-2.5.7; B:-2.5.6; ***C:+2.3.3; D:+2.5.7; E:+3.6.10.***



≅

1	1	2	3	3	3		u_i	k	s_i
3	**4**	**6**	**1**	**2**	**5**	***i***	AB**CDE**		123
0	**D**	–B	**C**	**C**	**C**	**3**	01**310**	1	103
	0	–B	**C**	**C**	**C**	**4**	01**310**	1	103
		0	**E**	**E**	**E**	**6**	02**003**	2	003
			0	–A	–A	**1**	20**201**	3	210
				0	–A	**2**	20**201**	3	210
					0	**5**	20**201**	3	210

Explanation: The vertices have ***three positions*** and vertex pairs are in ***five positions***, including *three "edge" positions* (full line ***C***, broken line ***E*** and broken line ***D***) and *two "non-edge" positions.*

For studying the structural properties is necessary *knowing the basic binary signs*, understand their meaning, ability *to read the model* **SM** *as a text of graph structure*.

Propositions 1.2. The ***basic binary signs*** $\pm d.n.q._{ij}$ describes the conditions of vertex pairs in the structure:

P1.2.1. Binary sign in the form $-d.n.q._{ij} = -\infty.2.0$ signify a ***disconnected vertex pair.***

P1.2.2. Binary sign in the form $-d.n.q._{ij}$ signify that the vertex pair forms a ***simple path*** or their *assemblage* with length ***d*** and we call it ***path sign.***

For example: *–2.3.2* shows a 2-path; *–3.4.3* shows a 3-path; *–7.8.7* shows a 7-path, etc.

P1.2.3. Binary sign with greatest absolute value $max|-d|$ show the ***diameter*** of the structure.

P1.2.4. Binary sign ***+d.n.q.$_{ij}$=+1.2.1*** signify that the vertex pair forms a ***link of a branch*** and we call it ***branch sign.***

P1.2.5. Binary sign ***+d.n.q.$_{ij}$***, where ***+d≥2,*** signify that the vertex pair belong to ***girth*** or their *assemblage* with length ***d+1*** and it is called ***girth sign.***

For example, girth signs are, ***+3.4.4*** for 4-girth; ***+4.5.5*** for 5-girth; …***+7.8.8*** for 8-girth etc. If the length ***+d*** no correspond with the numbers of vertices ***n*** and edges ***q***, then is touch with any mutually intercrossed *(d+1)-girths*.

P1.2.6. Binary sign in the form ***(+d=2).n.(q=n(n–1):2)*** signify belonging to a ***clique*** and we call it ***clique sign.***

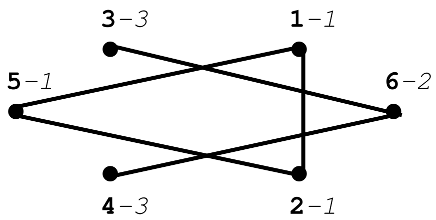
For example: ***+1.2.1*** show a 2-clique; ***+2.3.3*** – 3-clique; ***+2.4.6*** – 4-clique; ***+2.5.10*** – 5-clique; ***+2.6.15*** – 6-clique; …, ***+2.13.78*** – 13-clique etc. Clique sign is a ***complete invariant*** of the clique, i.e. to clique sign correspond only a clique.

It is useful treat also the ***accompanying graphs*** of a graph, such as *complement, pair graphs, sign graphs* and *adjacent graphs.*

The structure be studied (investigates) *in an integrated way*, in conjunction with its *complement*.

Example 1.3. The ***complement*** $\neg$***G*** of graph ***G*** (on example 1.2) and its ***model*** **SM**:

A:-2.3.2; B:-0.2.0; ***C:+1.2.1; D:+2.3.3.***



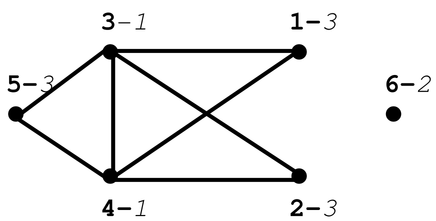
1	1	1	2	3	3		u_i	k	
1	**2**	**5**	**6**	**3**	**4**	*i*	ABCD		123
0	**D**	**D**	−B	−B	−B	**1**	03**02**	1	200
	0	**D**	−B	−B	−B	**2**	03**02**	1	200
		0	−B	−B	−B	**5**	03**02**	1	200
			0	**C**	**C**	**6**	03**20**	2	002
				0	−A	**3**	13**10**	3	010
					0	**4**	13**10**	3	010

Explanations: **a)** The ***vertex and binary positions*** in ***G*** and its *complement* $\neg$***G remain always.*** **b)** But if ***G*** has *three edge and two non-edge positions* then in complement $\neg$***G*** is it in *contrary.*

Binary graph be characterize the condition of a vertex pair, *binary sign* is only its invariant. In some cases, it is necessary to open the *structure model of binary graph* for adjustment of corresponding binary sign.

Example 1.4. ***Binary graph*** $g_{3.4}$ of the vertex pair 3-4 ***(D)*** of ***G*** and its ***model*** **SM:**

A:-2.4.5; B:-0.2.0; ***C:+2.3.3; E:+2.5.7.***



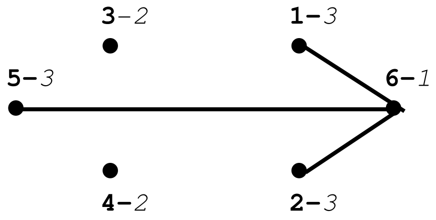
1	1	2	3	3	3		u_i	k	
3	**4**	**6**	**1**	**2**	**5**	*i*	ABCD		123
0	**D**	−B	**C**	**C**	**C**	**3**	01**31**	1	103
	0	−B	**C**	**C**	**C**	**4**	01**31**	1	103
		0	−B	−B	−B	**6**	05**00**	2	000
			0	−A	−A	**1**	21**20**	3	200
				0	−A	**2**	21**20**	3	200
					0	**5**	21**20**	3	200

Explanation: The ***elements***- and ***binary positions*** of ***G*** and its binary graph $g_{3.4}$ ***remain*** but this is not regular.

Sign graph is one of the key attributes of structure, by it can *adjust the pair signs* (Prop. 1.3) and investigate the *structural properties*. In some cases it may be turn out to the *position structure* (Prop. 2.9).

Example 1.5. ***Sign graph*** $\boldsymbol{G_{p=+E}}$ of by sign ***E: +3.6.10*** of ***G*** and its ***model*** **SM:**

A: -2.3.2; B: -0.2.0; ***C: +1.2.1.***



1	2	2	3	3	3		u_i	k	
6	**3**	**4**	**1**	**2**	**5**	***i***	*AB**C***		123
0	-B	-B	**C**	**C**	**C**	**6**	02**3**	1	003
	0	-B	-B	-B	-B	**3**	05**0**	2	000
		0	-B	-B	**-**B	**4**	05**0**	2	000
			0	-A	-A	**1**	22**1**	3	100
				0	-A	**2**	22**1**	3	100
					0	**5**	22**1**	3	100

Explanations: **a)** Sign graphs $\boldsymbol{G_{p=+C}}$ $\boldsymbol{G_{p=+D}}$ and $\boldsymbol{G_{p=+E}}$ of ***G*** are its *partial graphs*, whereby $\boldsymbol{G_{p=+C}}$ cover ***G*** completely and $\boldsymbol{G_{p=+D}}$ only vertex pair 3-4. **b)** Sign graphs $\boldsymbol{G_{p=\,--A}}$ and $\boldsymbol{G_{p=\,--B}}$ are partial graphs of complement ⅂***G***. **c)** Also here the ***positions remain*** but this is not regular.

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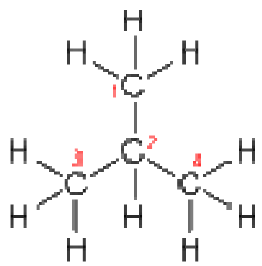
Structural models are useful also for investigation of ***natural objects***. For example, structural model of the *chemical compound* is a detailed presentation of the classical structural formula, i.e. of a *graph* that represents this formula. Structural model recognizes all the relationships between elements and their positions.

Example 1.6. Structural formula (i.e. graph) of *isobutan* C_4H_{10}, its binary signs and structural model:



A:-4.5.4; B:-3.4.3; C:-2.3.2;
D:+1.2.1.

2	1	3	4	11	5	6	7	8	9	10	12	13	14		***i***	u_i		s_i
C	**C**	**C**	**C**	**H**	**H**	**H**	**H**	**H**	**H**	**H**	**H**	**H**	**H**	***a***		*ABC**D***	***k***	1234
0	***D***	***D***	***D***	***D***	-C	-C	-C	-C	-C	-C	-C	-C	-C	**C**	2	009**4**	1	0**31**0
	0	-C	-C	-C	***D***	***D***	***D***	-B	-B	-B	-B	-B	-B	**C**	1	063**4**	2	**1**00**3**
		0	-C	-C	-B	-B	-B	***D***	***D***	***D***	-B	-B	-B	**C**	3	063**4**	2	**1**00**3**
			0	-C	-B	-B	-B	-B	-B	-B	***D***	***D***	***D***	**C**	4	063**4**	2	**1**00**3**
				0	-B	-B	-B	-B	-B	-B	-B	-B	-B	**H**	11	093**1**	3	**1**000
					0	-C	-C	-A	-A	-A	-A	-A	-A	**H**	5	633**1**	4	0**1**00
						0	-C	-A	-A	-A	-A	-A	-A	**H**	6	633**1**	4	0**1**00
							0	-A	-A	-A	-A	-A	-A	**H**	7	633**1**	4	0**1**00
								0	-C	-C	-A	-A	-A	**H**	8	633**1**	4	0**1**00
									0	-C	-A	-A	-A	**H**	9	633**1**	4	0**1**00
										0	-A	-A	-A	**H**	10	633**1**	4	0**1**00
											0	-C	-C	**H**	12	633**1**	4	0**1**00
												0	-C	**H**	13	633**1**	4	0**1**00
													0	**H**	14	633**1**	4	0**1**00

Explanation: Decomposition of the elements C and H to four positions is true.

This is a so-called systemic approach to the study of chemical compounds where different chemical elements (atoms) as a rule, are divided into different *positions* and can be treated as *subsystems*. In case of more complex compounds, however, may also the same kind of elements (atoms) belong to different positions (for example, ethanol, butane, propane, etc.). The main idea of using the models consists in treatment of the whol,e on the basis of positions and the relationships between them. Structural models open up the possibility for additional investigation of chemical compounds. The models of some polymers and organic matters tend to be very large [40]. Here be limited with a moderate.

Structural model allows to recognize: 1) *the structure of graph*, where an essential meaning have the *binary- and vertex- (elements-) positions* as well as the *sign- and position structures*; 2) *equivalence of the structures* and *isomorphism of the corresponding graphs;* 3) the elementary *structural transformations* in the form of *adjacent structures*.

Sign structure is composed of the element pairs that have the same basic binary signs. ***Position structure*** consists of element pairs that belong to the same position (Def. 2.3). ***Adjacent structure*** is a greatest substructure or smallest superstructure of a structure which is obtained with removing or adding a connection (edge) in a binary position (Def. 4.1). This is related with the ***reconstruction problem*** (Ulam Conjecture) and opens a way to formation of the ***systems of adjacent structures*** i.e. *systems of non-isomorphic graphs with **n** vertices*.

1.3. Adjustment and simplification of the model

In common case are the structures recognizable on the level of basic binary signs, but in case of some symmetric graphs is necessary to use the ***adjusted binary signs***. In the case of solving some practical tasks is suitable the binary signs ***to simplify***.

Adjustment

It is obvious that basic binary signs in the form ***d.n.q.*** does not always become to complete identifier of vertex pairs. To exact ascertaining the structure of s*ome large transitive and symmetric* graphs is needful to ***adjust the binary signs*** or to ***deep-identify*** [37]. For this be exist any possibilities. Also here assist the ***binary-*** and ***sign graphs.***

Propositions 1.3. Possibilities for ***adjust identifications***:

P1.3.1. Using complementary pair signs $\boldsymbol{d.n.q.}_{ij}^{m}$ of the ***high degree m binary graphs*** $\boldsymbol{g}_{ij}^{m}$. We call it ***high identification.***

For example, second degree binary graph $\boldsymbol{g}_{ij}^{m=2}$ is this, which remain between vertices ***i*** and ***j*** of ***G*** after removing the preliminary binary graph $\boldsymbol{g}_{ij}$, i.e. $\boldsymbol{g}_{ij}^{m=2} = \boldsymbol{G} \setminus [\boldsymbol{g}_{ij} \setminus (\boldsymbol{v}_i \& \boldsymbol{v}_j)]$.

P1.3.2. Using complementary binary signs of the ***local structure model*** $\mathbf{SM}_{ij}$ of first or high degree ***binary graphs*** $\boldsymbol{g}_{ij}$. We call it ***local identification.***

P1.3.3. Using complementary binary signs of the ***structure model*** $\mathbf{SM}_p$ of a ***sign graph*** $\boldsymbol{G}_p$. Such deep identification mode we call ***sign graph identification.***

As well, with multiplying (exponenting) the adjacency matrix ***E*** to a certain degree $\boldsymbol{E}^n$ increases *the values of its elements* as well as *the number of different values*. Increase takes place up to a degree ***n***, after which it stops. It turned out that these values identify the element pairs with exactness up to the binary positions (and on their basis the positions of the elements) [33]. We call corresponding values to *productive binary signs*.

Productive Identification Algorithm *(PIA)*. OPERAND: *List of adjacent elements* ***L.*** ALGORITHM: 1) Form the adjacency matrix ***E.*** 2) Multiple it with itself $\boldsymbol{E}\times\boldsymbol{E}\times\boldsymbol{E}\times\ldots= \boldsymbol{E}^n$ and fix in case of each degree ***n*** the number ***p*** of obtained differences among *productive binary signs* $\boldsymbol{e}^{n}_{ij}$, which as rule enlarge. 3) If ***p*** more no enlarge, then to stop the multiplication and to fix the last product $\boldsymbol{E}^n$, corresponding ***p*** and successive $\boldsymbol{E}^{n+1}$. 4) Obtained matrix products $\boldsymbol{E}^n$ and $\boldsymbol{E}^{n+1}$, as certain types "sign matrices". 5) If necessary, to specify the binary signs $\pm\boldsymbol{d.n.q.}_{ij}$ with the productive $\pm\boldsymbol{d.n.q.e}^{n}\boldsymbol{.}_{ij}$. RESULT: An adjusted model **SM*.**

Explanations: **a)** The maximum number ***p*** can be greater than the number of differences among the basic binary signs and in some cases to *adjust* these. **b)** Adjustably identified binary sign, for example ±***d.n.q.e***^{n}**.**$_{ij}$ we call to ***adjusted sign*** and corresponding model to ***adjusted model*** **SM*** (example 3.6).

Example 1.7. Result of product identification ***PIA*****:** The product ***E***x***E***x***E=E***^{3} of adjacency matrices and adequacy of basic and productive binary signs of a transitive graph**:**

Basic binary signs	*0*	*-2.6.11*	***+2.5.8***	***+2.4.5***	***+2.4.6***
Productive binary signs e_{ij}^3	*12*	*13*	***16***	***18***	***19***

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**=i**	**u$_i$=12345**	**k**
12	***18***	***16***	*13*	***19***	*13*	***16***	***18***	1	*12*221	*1*
18	*12*	***18***	***16***	*13*	***19***	*13*	***16***	2	*12*221	*1*
16	***18***	*12*	***18***	***16***	*13*	***19***	*13*	3	*12*221	*1*
13	***16***	***18***	*12*	***18***	***16***	*13*	***19***	4	*12*221	*1*
19	*13*	***16***	***18***	*12*	***18***	***16***	*13*	5	*12*221	*1*
13	***19***	*13*	***16***	***18***	*12*	***18***	***16***	6	*12*221	*1*
16	*13*	***19***	*13*	***16***	***18***	*12*	***18***	7	*12*221	*1*
18	***16***	*13*	***19***	*13*	***16***	***18***	*12*	8	*12*221	*1*

Explanations: **a)** In present case is the complete identify attainable on the level of the third degree e_{ij}^3. **b)** Productive binary signs no contain direct structural data.

In principle the structure's model could be based only on the productive binary signs, if would be know what these mean. It is only said that the elements of E^n characterize the longest paths (chains) between structural elements. Unfortunately, it is questionable because the values exist also on the main diagonal, at the same time as in elsewhere the values are also at times turn out to be zero. These no distinguish the adjacent and non-adjacent elements but are well suited to refine the basic binary signs.

Apparently no one has interested in the question: *why detect the elements of obtained matrix* E^n *the binary positions?* Already in 1976 was drawn attention to the too one-sided approach to graphs which hinders the development of graph theory [18].

The *basic binary signs* not lose its meaning, these characterizes the relationships between vertices, the belonging of vertex pairs to (assemblage of) paths or girths with corresponding size etc. These are needed for characterizing of the structure as a whole. In case of some strongly regular graphs where need use the *local identification* method P1.3.2.

Simplification

We had afore shown rather symmetric graphs. For all the graphs generally be valid follow proposition.

Proposition 1.4. Almost all the graphs are ***0-symmetric, connected*** and with ***diameter 2.***

This means, that each vertex and vertex pair constitute a single position in structure. Be lacking any symmetry, the number of pair signs is very large. For solving applicative tasks is necessity to find some ***"similarity"*** between elements.

Since real ***communication networks*** are very large. Imagine here one a peculiar companionship ***Z*** consisting of *Adolf, Birgit, Charles, Diana, Erik, Frieda, George, Helen, Ingvar and Jane.* They are mutually agreed that everyone communicates with the five, known to us, parlor companions. The latter circumstance had required of coordination, and someone had to do it.

This situation constitutes a *five-degree-regular structure* in which all the members seem to be in "equal position" [39].

Example 1.8. To present this situation make a corresponding model ***Z***:

A:-2.6.10; B:-2.6.9; C:-2.5.8; D:-2.5.7; E:-2.5.6; F:-2.4.5; G:-2.4.4;
H:+2.3.3; I:+2.4.5; J:+2.5.7; K:+3.10.25.

1	2	3	4	5	6	7	8	9	10		u_i	
F	**A**	**D**	**H**	**C**	**B**	**I**	**J**	**E**	**G**	*name*	*ABCDEFG**HIJK***	*k*
0	-G	**I**	**J**	-D	-F	**I**	-E	**I**	**H**	**Frieda**	0001111**1310**	1
	0	**H**	-G	**J**	**I**	-D	**I**	-D	**I**	**Adolf**	0002002**1310**	2
		0	-C	**I**	-D	**H**	-E	**I**	-D	**Diana**	0012100**2300**	3
			0	-E	**H**	**I**	-B	**H**	**H**	**Helen**	0110101**3110**	4
				0	**H**	-G	-A	**H**	**H**	**Charles**	1001101**3110**	5
					0	**I**	**I**	-E	-A	**Birgit**	1001110**2300**	6
						0	**H**	-A	-D	**Ingvar**	1002001**2300**	7
							0	**K**	**H**	**Jane**	1100200**2201**	8
								0	-B	**Erik**	1101100**2201**	9
									0	**George**	1102000**4100**	10

The structure ***Z*** is *0-symmetric*, there do not "equality", each member has its own private position. *Different position* means different connectivity, "relationships" with other members. Between ten members exists 11 different relationships, which is characterized by the binary signs (see frequency vectors $\boldsymbol{u}_i$). The problem lies here in the *grouping of strictly differentiated members*. This fact leads us back to the *sign structures* $\boldsymbol{GS}_p$. In selection of the sign must be proceeds from:

1) Selected sign must be exists *in case of each structural element*.
2) To keep in mind the *meaning of sign*, because the sign structure be formed on the aspect of sign.

In principle is the companionship decomposable to the eleven inseparable component sign structures $\boldsymbol{GS}_p$, and gives different groupings. This is inappropriate, and useful to go the other way.

Let to it is the rearranging the members by their "direct communication signs" ***HIJK*** of frequency-vectors.

Example 1.9. Rearranged by ***HIJK*** model ***Z***:

1	2	4	5	3	6	7	8	9	10			u_i	
F	**A**	**H**	**C**	**D**	**B**	**I**	**J**	**E**	**G**	*name*	*k*	***HIJK***	**R**
0	-G	**J**	-D	**I**	-F	**I**	-E	**I**	**H**	**Frieda**	1	***1310***	**1**
	0	-G	**J**	**H**	**I**	-D	**I**	-D	**I**	**Adolf**	2	***1310***	**1**
		0	-E	**-C**	**H**	**I**	-B	**H**	**H**	**Helen**	4	***3110***	**2**
			0	**I**	**H**	-G	-A	**H**	**H**	**Charles**	5	***3110***	**2**
				0	-D	**H**	-E	**I**	**-D**	**Diana**	3	***2300***	**3**
					0	**I**	**I**	-E	-A	**Birgit**	6	***2300***	**3**
						0	**H**	-A	-D	**Ingvar**	7	***2300***	**3**
							0	**K**	**H**	**Jane**	8	***2201***	**4**
								0	-B	**Erik**	9	***2201***	**4**
									0	**George**	10	***4100***	**5**

The resulting grouping corresponds to the requirement of "direct communication signs", where the *ten positions* ***k*** reduces to *five groups*, with the members:

R_1= (*Frieda, Adolf*), R_2= (*Helen, Charles*), R_3= (*Diana, Birgit, Ingvar*),
R_4= (*Jane, Erik*) and R_5= (*George*).

For finding the "similarity" of members can be use also *approximate or rounded-off* binary signs.

Example 1.10. Using the rounded-off binary signs:

Rounding-off: *a*= [*A:-2.6.10; B:-2.6.9*], *b*= [*C:-2.5.8; D:-2.5.7; E:-2.5.6*],
c= [*F:-2.4.5; G:-2.4.4*], **d= [*H: +2.3.3; I:+2.4.5; J: +2.5.7*], e= (*K: +3.10.25*).**
Rounded binary signs: *a*:(*A, B*) ≈ –2.6, *b*:(*C, D, E*) ≈ –2.5, *c*:(*F, G*) ≈ –2.4, ***d*:(*H, I, J*) ≈ +2** ja ***e*: *K* ≈ +3.**

1		2	3			4		5			a	b	c	**d**	**e**		**u*ᵢ**	
F	**A**	**D**	**H**	**C**	**B**	**I**	**J**	**E**	**G**	***name***	*AB*	*CDE*	*FG*	***HIJ***	***K***	***k***	*abc****de***	***k****
0	*-G*	***I***	***J***	*-D*	*-F*	***I***	*-E*	***I***	***H***	**Frieda**	*00*	*011*	*11*	***131***	***0***	1	*022****50***	***1***
	0	***H***	*-G*	***J***	***I***	*-D*	***I***	*-D*	***I***	**Adolf**	*00*	*020*	*02*	***131***	***0***	2	*022****50***	***1***
		0	*-C*	***I***	*-D*	***H***	*-E*	***I***	*-D*	**Diana**	*00*	*121*	*00*	***230***	***0***	3	*030****50***	***2***
			0	*-E*	***H***	***I***	*-B*	***H***	***H***	**Helen**	*01*	*101*	*01*	***311***	***0***	4	*121****50***	***3***
				0	***H***	*-G*	*-A*	***H***	***H***	**Charles**	*10*	*011*	*01*	***311***	***0***	5	*121****50***	***3***
					0	***I***	***I***	*-E*	*-A*	**Birgit**	*10*	*011*	*10*	***230***	***0***	6	*121****50***	***3***
						0	***H***	*-A*	*-D*	**Ingvar**	*10*	*020*	*01*	***230***	***0***	7	*121****50***	***3***
							0	***K***	***H***	**Jane**	*11*	*002*	*00*	***220***	***1***	8	*220****41***	***4***
								0	*-B*	**Erik**	*11*	*011*	*00*	***220***	***1***	9	*220****41***	***4***
									0	**George**	*11*	*020*	*00*	***410***	***0***	10	*220****50***	***5***

The resulting grouping by rounded-off binary signs:

k^*_1= (*Frieda, Adolf*), k^*_2= (*Diana*), k^*_3= (*Helen, Charles, Birgit, Ingvar*),
k^*_4= (*Jane, Erik*) and k^*_5= (*George*).

We can see that there exist coincidences between the results of "direct communication signs" and rounding-off. The first way shall be considered as more distinct and therefore more reliable. The "rounding" of binary signs may prove to be quite arbitrary. Here can remark a *specific role of Jane and Erik* in this companionship, to their relationship ***K: +3.10.25*** includes all members and relationships, and they may be *coordinators.*

Such *0*-symmetric structures can be treats, investigate, and elements grouped in several ways:

1) By investigation of the selected sign structures GS_p.
2) By investigate on the basis of some selected binary signs formed the so-called complex sign structures.
3) By reordering the structural model by the given binary signs (example 1.9).
4) For reducing the positions to use the connected or "rounded" binary signs (example 1.10).

All of this requires a good knowledge of the subject and suitable choices the aspects for the investigation.